



Achieving Mechanical Versatility in Robots and Structures Through Laminar Jamming

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Achieving Mechanical Versatility in Robots and Structures Through Laminar Jamming

A dissertation presented

by

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to

The Harvard John A. Paulson School of Engineering and Applied Sciences

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Abstract

There are two major physical paradigms in robotics—soft robots and traditional rigid robots. Soft robots are made of compliant materials and have excellent adaptivity, robustness, and safety, whereas traditional rigid robots are made of stiff materials and have outstanding resolution, precision, speed, and load capacity. Building a single system that can selectively behave like either a soft or traditional rigid robot has been a grand challenge of the field.

In this thesis, we rigorously investigate a promising mechanism that can help unite these paradigms. The mechanism is laminar jamming, in which a stack of flexible layers can exhibit dramatic changes in its mechanical properties (e.g., stiffness) when a pressure gradient is applied. When laminar jamming structures are integrated into soft robots, they can begin to exhibit the form, and consequently the function of traditional rigid systems.

The mechanism was first reported in the robotics literature in 2000 [1]. However, a surprising number of fundamental questions have been left unexplored: For instance, what is the physical mechanism behind the phenomenon? How can the deformation of laminar jamming structures be predicted for both small and large loads, as well as during dynamic motions? Beyond stiffness, what other mechanical properties can the phenomenon change?

In this thesis, we demonstrate how the laminar jamming phenomenon works; how laminar jamming structures can transform the stiffness, damping, kinematics, and dynamic response of robotic systems; how designers can relate design parameters to performance metrics; and how the performance of laminar jamming structures can be pushed well past the state-of-the-art. In doing so, we aim to foster robots and structures that cannot simply be classified as “soft” or “rigid,” but instead exhibit highly versatile mechanical behavior.

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Chapter 1

Introduction

The physical paradigms of traditional rigid robotics and soft robotics are highly disparate in form and function. How do we build devices that can selectively exhibit the behavior of either? One answer is to endow robotic systems with mechanical versatility—in other words, the ability to rapidly change their fundamental mechanical properties (e.g., stiffness and damping) on command. This thesis deeply explores a less-explored physical phenomenon, laminar jamming, that allows systems to manifest this versatile behavior.

1.1 Motivation

Traditional rigid robots typically consist of rigid links connected by a small number of revolute or prismatic joints, with an end-effector located on the most distal link [2]. The links are fabricated from metals with elastic moduli on the order of $10 - 100 GPa$, and the joints are often actuated by electric motors or high-pressure hydraulic actuators. Development of these systems has been successfully translated to industry; current examples of products include heavy-duty robotic arms made by FANUC [3], Yaskawa Motoman [4], ABB [5], Kawasaki [6], and KUKA [7]; light-duty, low-cost robotic arms made by ABB (e.g., YuMi [8]), Universal Robots [9], and Rethink Robotics [10]; and robotic hands made by Barrett Technology [11] and Shadow Robot Company [12].

Deformation of traditional rigid robots typically occurs at the joints alone, and actuators

control the angular or translational positions of these joints. The relationship between actuator inputs and the configuration of the robots can be effectively described using trigonometry; thus, modeling of these systems, particularly serial manipulators, can often be readily performed. Even when interacting with objects, the systems are frequently assumed to be position sources, with configurations that are not altered during interaction.

On the other hand, soft robots typically consist of flexible open or closed surfaces that neither have well-defined joints nor end-effectors. The surfaces are often fabricated from rubber, plastic, or gel with small-strain stress-strain-moduli on the order of $100kPa - 100MPa$. The surfaces are often actuated by low-pressure pneumatic or hydraulic chambers or dielectric-elastomer actuators. Notable examples of these systems are fluid-powered rubber rippers [13–17] and walking robots [18, 19], and excellent reviews of soft robots are provided in [20] and [21]. Development of these systems is just starting to be translated to industry [17].

Unlike with traditional rigid robots, deformation of soft robots typically occurs along their entire surface or body, and actuators are commonly under pressure or voltage control, rather than position control. Thus, the relationship between actuator inputs and the configuration of the robots cannot be effectively described using trigonometry, but requires the mathematical methods of continuum mechanics (e.g., solid mechanics, fluid mechanics, and fluid-structure interaction). Thus, modeling of these systems is typically much more challenging than for traditional rigid robots. Moreover, when interacting with objects, these systems can rarely be approximated as position sources due to their intrinsic compliance, making their configuration during interaction particularly challenging to predict. Whereas biological analogues to traditional rigid robots include the human arm and hand, analogues to soft robots are less anthropomorphic—for example, octopus tentacles and elephant trunks, both of which are compliant, but highly dexterous.

The distinct physical forms of traditional rigid robots and soft robots, as well as modeling and control disparities, lead to distinct functions. Rigid robots are well-suited to perform tasks that require heavy lifting, rapid motion, and precise manipulation, such as pick-and-place operations in factories and cutting operations in surgery [22]. On the contrary, soft robots are

well-suited for tasks that demand safe interaction with humans, adaptivity to the environment, or manipulation of delicate objects, such as human gait augmentation [23] or handling of fresh produce [17].

This thesis has been motivated by the question, “Can we build a single robotic system that can behave like either a soft robot or a traditional rigid robot on command?” This capability could foster a new generation of highly versatile machines. A few examples are

- A robotic hand that can perform wrap grasps around cylindrical, spherical, or amorphous objects (e.g., coffee mugs, basketballs, or rocks) in a compliant, jointless state, and execute pinch grasps on thin or flat objects (e.g., pencils or keys) in a rigid, jointed state
- Intelligent knee braces that can detect gait events, tune stiffness and damping to reduce muscle activation and metabolic energy expenditure, and ensure safe recovery from adverse occurrences (e.g., stumbles)
- A multigait locomotion robot that can crawl over rugged terrain in a compliant, jointless state, and efficiently walk upright on flat ground in a rigid, jointed state
- A swimming robot that can tune its body stiffness (and thus, its hydroelastic resonant frequency) to maximize energy efficiency of forward propulsion at multiple speeds

In between the paradigms of soft robotics and traditional rigid robotics, compelling systems have also been made that combine both soft and rigid elements (e.g., robotic hands with rigid links and compliant flexure joints [24, 25]). These systems can compliantly adapt to their environment and apply high directed forces. However, since these systems include both soft and rigid elements, they are limited in both their compliance and rigidity; furthermore, they cannot dynamically transition between a soft robotic state and a traditional rigid robotic state.

To tackle the question of whether a single system can behave like a soft robot or traditional rigid robot on command, it is worthwhile to dissect “soft” and “rigid” behavior into their constituent parts. From a mechanical perspective, there are three fundamental differences between soft robots and traditional rigid robots: stiffness, damping, and kinematics. Specifically,

soft robots typically have lower stiffness than traditional rigid robots, have higher passive damping (often due to the viscoelasticity of soft materials, such as castable urethane), and do not typically have joints. A robot that can transition between a soft robotic state and a traditional rigid robotic state should have a mechanism to reversibly alter these specific parameters.

This thesis rigorously investigates a candidate mechanism that may be able to accomplish this goal. The mechanism is laminar jamming (also known in the robotics literature as “layer jamming” or “sheet jamming”), in which a stack of flexible layers exhibits dramatic changes in its mechanical properties on application of an external pressure gradient (i.e., a pressure differential between the pressure acting on the layers and ambient pressure). As will be shown, laminar jamming structures can rapidly and reversibly transform stiffness, damping, and kinematics; furthermore, they can be readily integrated into soft robots, endowing them with the ability to reversibly behave like traditional rigid robots. We now proceed to review other candidate mechanisms and explain why laminar jamming was chosen for investigation.

1.2 Background

1.2.1 Variable-Impedance Mechanisms

In electrical circuits, common circuit elements (e.g., capacitances and inductors) have a frequency-dependent resistance and phase. To make mathematical analysis of these circuits convenient, circuit elements are associated with a complex impedance, from which both resistance and phase can be extracted. Analogous to electrical impedances, the stiffness and damping values of springs and dampers are often referred to in the robotics literature as “mechanical impedance,” and tunable-stiffness and tunable-damping mechanisms are referred to as “variable-impedance mechanisms.”

An excellent review of variable-impedance actuators is presented in [26]. The review proposed a classification of variable-impedance actuators into three major categories: active impedance by control, inherent compliance, and inherent damping. Inspired by this classifi-

cation, we propose that variable-impedance mechanisms (which include both actuators and structural elements) may be divided into two major categories:

1. *Extrinsic variable-impedance mechanisms*, in which the intrinsic stiffness and damping of the mechanism is negligible, and the stiffness and damping are instead continuously controlled. No energy is stored in the process. As an example, consider an electric motor. Let the intrinsic stiffness and damping of the motor be negligible. Assume that the motor has a joint encoder that measures angular error with respect to a predefined equilibrium value. The error can be fed back through a proportional-derivative controller to generate a current (i.e., torque) command, resulting in a non-negligible resistance to motion that is equivalent to that of a spring and damper. No energy is stored in the form of elastic energy.
2. *Intrinsic variable-impedance mechanisms*, in which a constituent material or a structure has an intrinsic stiffness and damping that is non-negligible, and the stiffness and damping can be changed upon application of a stimulus. Energy can be stored in the material. As an example, consider a low-melting-point metallic alloy, such as solder or Field’s metal. In the solid state, the alloy has a high intrinsic stiffness and negligible damping, and as the alloy deforms, energy can be stored as elastic energy. Upon heating, the solid changes phase to a liquid, which has a negligible intrinsic stiffness and non-negligible viscous damping.

In this research, only candidate mechanisms from the second category were considered. These mechanisms are not reliant on continuous sensing and control, which results in improved bandwidth (e.g., for impact responses); furthermore, these mechanisms allow energy storage, which is critical for energetic efficiency (e.g., in walking robots).

Variable-Stiffness Mechanisms

Thorough reviews of intrinsic variable-stiffness mechanisms are presented in [27, 28]. Examples include antagonistic actuation mechanisms, which can consist of fluidic chambers (pushing

outward) in parallel with thin metallic tendons (pulling inward) [29]; magnetorheological (MR) fluids [30], which are non-Newtonian fluids that can be approximated as Bingham plastics, with a yield stress that is a function of the applied magnetic field; and phase-change materials (e.g., [31–33]), which include low-temperature metallic alloys described earlier.

As described in [28], the advantages of laminar jamming over other variable-stiffness mechanisms are its high speed of actuation (particularly in comparison to thermally-actuated mechanisms), high scalability of dimensions, and independent control of stiffness and position. However, [28] specifies that laminar jamming can achieve a stiffness increase of up to a factor of 10, which is a notable underestimate. In fact, as explained later, the stiffness increase scales with n^2 , where n is the number of layers in the laminar jamming structure; thus, stiffness changes of > 1000 can be achieved with just 32 layers.

Variable-Damping Mechanisms

A review of intrinsic variable-damping mechanisms is included in the previously-cited review on variable-impedance actuators [26]. Examples include MR fluids, which dissipate energy after yield (i.e., during flow); Eddy-current mechanisms, in which a magnetic field dissipates energy in a moving conductor; and linear fluidic mechanisms (e.g., dampers on garage doors), which dissipate energy during flow and provide a resistive force approximately proportional to velocity. The magnitude of energy dissipation in an MR fluid and Eddy-current mechanism can be rapidly altered by tuning the magnetic field, whereas energy dissipation in a linear fluidic mechanism can be altered by dynamically tuning the cross-sectional area of the orifice through which the fluid flows.

Laminar jamming is not cited as a variable-damping mechanism in [26]. However, laminar jamming structures act as a form of friction dampers, which are presented and critiqued in the review. The authors state that the tunability of damping is hampered by the challenge of dynamically altering friction coefficients; however, as discussed later, this problem can be readily overcome in laminar jamming structures by instead tuning the normal force acting on the layers of material (e.g., by adjusting the vacuum pressure acting on a laminar jamming

structure). By such means, damping in laminar jamming structures can be tuned rapidly with essentially infinite resolution. To our knowledge, only a single study in the robotics literature has used laminar jamming as a damping mechanism [34]; nevertheless, this study did not harness the intrinsic damping behavior of laminar jamming structures, but used feedback to actively convert them to viscous dampers with limited bandwidth.

1.2.2 Variable-Kinematics Mechanisms

As described earlier, soft robots and traditional rigid robots have disparate kinematics. Soft robots typically deform continuously along their surfaces or bodies, whereas traditional rigid robots deform exclusively at joints. To our knowledge, the only other studies that have enabled robots to rapidly transition between jointless and jointed states are [35, 36], which were contemporary with our research.

In these studies, a robotic finger was constructed consisting of an inflatable chamber on top of a stiff layer. In [35], the stiff layer consisted of rigid polylactic acid links connected by shape-memory polymer (SMP) at intended joint locations; in [36], the stiff layer consisted of an SMP matrix, with conductive thermoplastic urethane (TPU) heating elements at intended joint locations. In both cases, heat was applied to the intended joint locations, causing local softening and creating joints. Compared to the laminar jamming mechanism that this thesis proposes, the mechanism in [35, 36] is limited by high stiffness when no power is applied, slow transition times between the compliant and rigid states, and low energetic efficiencies.

1.2.3 Granular Jamming

The preceding review of variable-impedance structures has provided a rational basis to further investigate laminar jamming as a state-of-the-art variable-stiffness, variable-damping, and variable-kinematics mechanism. Nevertheless, before discussing laminar jamming in detail, we first briefly describe and differentiate a partial namesake, granular jamming (also known in the robotics literature as “particle jamming”).

In granular jamming, a disordered system of particles (e.g., sand or coffee grounds) can

transition from a liquid-like state to a solid-like state. For frictionless particles, jamming occurs when the packing density is increased past a critical value and the applied shear stress is sufficiently low, allowing geometric constraints between some sets of neighboring particles to prevent the flow of other particles. Unjamming occurs when the packing density increases or the applied shear stress is sufficiently high, causing dilation and yielding. The same processes are applicable to frictional particles; however, for a limited range of packing densities, jamming may also occur upon an *increase* in shear stress [37]. Accurate continuum models of granular jamming have long remained elusive, but recent breakthroughs have been made [38]. An excellent review of the mechanics and applications of granular jamming is provided in [37].

In robotics, granular jamming has been typically implemented by enclosing particles in a flexible, airtight envelope and applying a vacuum to the envelope to increase the packing density of the particles. Granular jamming structures have been used as variable-stiffness mechanisms in locomotion robots [39], haptic interfaces [40, 41], articulated manipulators [27, 42, 43], and most notably, grippers [44–46]. However, the performance of granular jamming structures is limited by their inability to withstand high tensile or bending loads due to the resulting separation between adjacent particles [42]. In standard materials, this separation is resisted by atomic bonds; however, in a granular jamming structure, the separation is typically resisted only by the mechanism that increases the packing density of the particles (e.g., vacuum), which tends to be weaker by many orders of magnitude.

As with granular jamming, laminar jamming structures exhibit dramatic changes in mechanical properties when an external pressure gradient is applied. Yet from both a mechanistic and practical point of view, laminar jamming and granular jamming are distinct. Mechanistically, laminar jamming is not a result of geometric constraints, it does not occur without the presence of friction or adhesion, it can occur without any change in packing density, and it cannot be activated by the application of shear stress. Moreover, the major practical advantage of laminar jamming over granular jamming is that laminar jamming structures can easily support bending loads, as elements can sustain tensile stresses without separation.

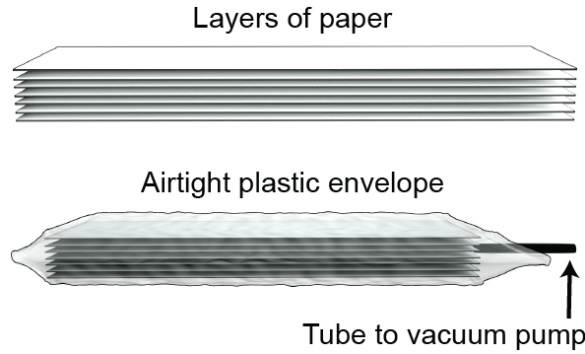


Figure 1.1: *Example of a vacuum-activated laminar jamming structure*

1.2.4 Laminar Jamming

As described earlier, laminar jamming structures consist of a stack of flexible layers to which an external pressure gradient is applied. In our implementation of laminar jamming, the flexible layers (e.g., strips of paper) are enclosed in a compliant, airtight envelope (e.g., a thin polyethylene or polyurethane bag), and the pressure gradient is imposed by connecting the envelope to a vacuum line (Figure 1.1). In contrast to physical implementations of other variable-impedance mechanisms, this form factor is thin, lightweight, low-cost, and easy-to-fabricate, facilitating rapid integration into existing robotic systems.

With vacuum off, the structure is highly compliant in bending, but with vacuum on, the structure is highly rigid in bending. The basic intuition behind the stiffness change is as follows:

- **When vacuum is off**, the layers experience no friction along their interfaces, and slip can freely occur during bending (i.e., points on adjacent layers that were initially coincident along an interface can move with respect to each other). Each layer bends in approximately the same fashion (Figure 1.2). Low tensile and compressive strains are generated within each layer, and the energetic cost of bending is low.
- **When vacuum is on**, the layers experience friction along their interfaces, and slip cannot freely occur during bending (i.e., points on adjacent layers that were initially coincident along an interface cannot move with respect to each other). The layers bend

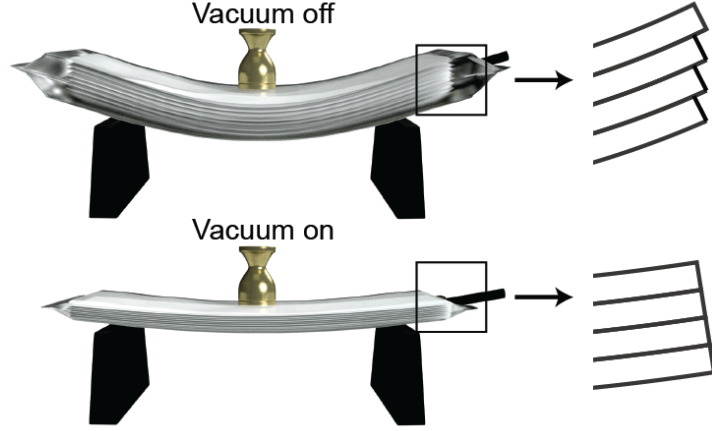


Figure 1.2: *A vacuum-activated laminar jamming structure under load*

as a single cohesive structure (Figure 1.2). High tensile and compressive strains are generated within the structure, and the energetic cost of bending is high.

For a slightly more rigorous explanation, approximate each layer as a thin beam. From classical Euler-Bernoulli beam theory (see [47] for an excellent visual introduction),

$$M = EI\kappa \quad (1.1)$$

where M is the resultant bending moment at a particular cross-section, E is the elastic modulus, I is the second moment of area (also known as the “area moment of inertia”), and κ is the curvature. The quantity EI is often referred to as the bending stiffness. For a single layer with a rectangular cross-section, $I \propto bh^3$, where b is the width of the layer and h is the height of the layer.

When no vacuum is applied, the layers behave like springs in parallel, and

$$I \propto nbh^3 \quad (1.2)$$

where n is the number of layers.

When vacuum is applied, the layers behave like a cohesive structure, and

$$I \propto b(nh)^3 = n^3bh^3 \quad (1.3)$$

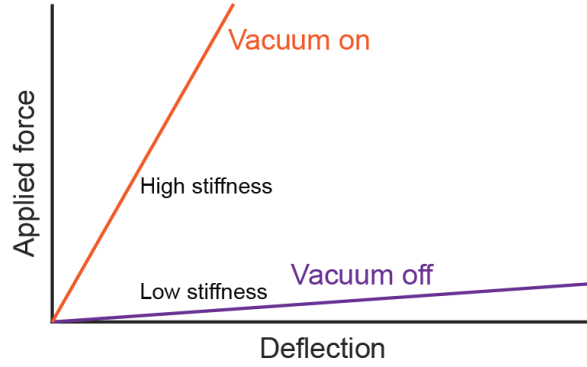


Figure 1.3: *Oversimplified force-deflection curves of laminar jamming structures in bending*

Thus, when vacuum is applied, the bending stiffness increases by a factor of n^2 (Figure 1.3).

Nevertheless, this stiffness change alone is an oversimplified representation of laminar jamming. In fact, in the vacuum-on state, when a certain critical load is exceeded, the layers begin to slip (i.e., points on adjacent layers that were initially coincident along an interface begin to move with respect to each other).

The basic intuition for this phenomenon as follows: a general transverse loading condition (e.g., 3-point bending) causes a resultant shear and resultant moment profile throughout the structure. In general, the moment varies with the longitudinal coordinate (i.e., the x -coordinate in Figure 1.4), which implies that the curvature, bending strain (i.e., tensile or compressive strain acting in the longitudinal direction), and bending stress also vary with the longitudinal coordinate. From static equilibrium (i.e., $\frac{\partial \sigma_{ij}}{\partial x_j} = 0$, where σ_{ij} denotes the second-rank stress tensor and $\frac{\partial}{\partial x_j}$ is the divergence operator), a bending stress that varies in the longitudinal direction induces a longitudinal shear stress that varies in the transverse direction (i.e., the y -coordinate in Figure 1.4). Thus, a transverse load induces a longitudinal shear stress, which ultimately causes the layers to slip.

More quantitatively, from classical Euler-Bernoulli beam theory, the maximum shear stress induced in the structure is equal to

$$\tau = \frac{3V}{2A} = \frac{3V}{2nbh} \quad (1.4)$$

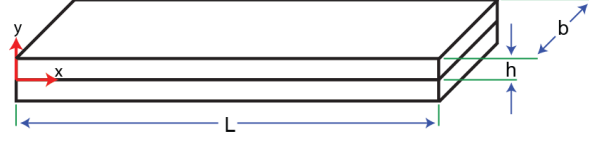


Figure 1.4: *Dimensional and coordinate conventions for a laminar jamming structure*

where τ is the shear stress, V is the resultant shear, and A is the cross-sectional area [48]. When the layers are coupled through vacuum, the maximum shear stress that can be sustained at any given interface is given by

$$\tau_{max} = \mu P \quad (1.5)$$

where τ_{max} is the maximum sustainable shear stress, μ is the coefficient of friction between the layers, and P is the vacuum pressure (defined here as the absolute pressure inside the envelope below ambient pressure). When the maximum induced shear stress equals the maximum sustainable shear stress, the layers begin to slip. Thus, slip occurs when

$$\frac{3V}{2nbh} = \mu P \quad (1.6)$$

For 3-point bending, $V = \frac{F}{2}$. Thus,

$$F_{crit} = \frac{4nbh\mu P}{3} \quad (1.7)$$

where F_{crit} is the critical force at which slip begins.

After slip initiates at a particular location on a particular interface, it gradually begins to occur at other locations and other interfaces as the load is increased. Ultimately, when slip has occurred at all possible locations along all interfaces, the structure provides no additional resistance to longitudinal shear, and the bending stiffness approximately decreases to that of the vacuum-off state. Furthermore, because of friction between the layers, energy is dissipated as slip occurs, and the structure begins to act as a damper. Thus, a more complete version of Figure 1.3 is shown in Figure 1.5. The precise details of this rich nonlinear behavior are explored in subsequent chapters, particularly Chapter 2.

To our knowledge, laminar jamming was first proposed in the robotics literature in 2000 [1].

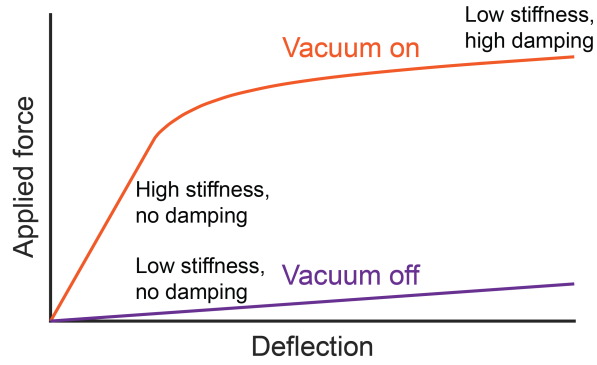


Figure 1.5: *Realistic force-deflection curves of laminar jamming structures in bending (derived from finite element simulations in [49])*

The study constructed a stack of layers of alternating electrodes and dielectrics (i.e., conductors and insulators), applied a voltage, and measured a change in bending stiffness. The study was followed by two publications in 2002 and 2003 [34, 50] in which vacuum-activated jamming structures were first implemented, and a correct derivation was provided for the n^2 scaling law that describes the increase in bending stiffness under small loads. Researchers then used laminar jamming structures as stiffening elements in multiple applications, including assistive devices [51, 52], surgical and articulated manipulators [53, 54], haptic interfaces [55–57], locomotion robots [58], and soft actuators [59].

Although multiple studies have been published on the concept and applications of laminar jamming, in fact, a tremendous amount of knowledge and innovation has been left unexplored. As examples, no studies provided analytical or computational models of laminar jamming beyond small loads (i.e., before the initiation of slip) or in dynamic motions; only one study considered laminar jamming as a damping mechanism (and the intrinsic friction-damping function was ignored [34]); no studies used laminar jamming to transform kinematics or dynamic response; and no studies considered using multiple materials in the same jamming structure in order to improve performance-to-mass ratios. This research has tackled these and other open questions, as well as showing how the answers can enable soft robots and structures to reversibly behave like traditional rigid systems.

1.3 Outline

The subsequent chapters of the thesis are constructed as follows:

1. **Chapter 2:** This chapter investigates the static behavior of laminar jamming structures and addresses the following questions:
 - (a) How does laminar jamming work?
 - (b) How can we analytically and computationally predict the quasi-static behavior of laminar jamming structures for small and large loads (i.e., before and after slip has initiated)?
 - (c) Can we provide researchers with scaling relations to design laminar jamming structures that can meet arbitrary performance requirements?
 - (d) How can laminar jamming enable soft robots to behave like traditional rigid robots in quasi-static applications (e.g., the ability to turn joints on and off in a robotic gripper)?
2. **Chapter 3:** This chapter explores the dynamic behavior of laminar jamming structures and addresses the following questions:
 - (a) How can we computationally predict the dynamic behavior of laminar jamming structures for small and large loads?
 - (b) Can we analytically approximate dynamic behavior with a simple discrete model that can provide designers with intuition?
 - (c) How can laminar jamming structures tune dynamic impact responses in soft structures (to make them respond like rigid ones) and rigid aerial robots (to make them respond like soft ones)?
3. **Chapter 4:** This chapter proposes jamming-based sandwich structures, a novel class of structures that crosses the performance boundaries of existing laminar jamming structures. It answers the following questions:

- (a) Can we improve the performance-per-weight of laminar jamming structures by using multiple materials in a sandwich configuration?
 - (b) Can we provide researchers with scaling relations to design jamming-based sandwich structures that can meet arbitrary performance requirements?
 - (c) Given practical constraints (e.g., mass and volume limitations), what is the best-possible laminar jamming structure or jamming-based sandwich structure that we can build?
 - (d) How can sandwich-based jamming structures improve the performance of assistive devices?
4. **Chapter 5::** This chapter summarizes the findings of this dissertation and proposes compelling opportunities for future research.

1.4 Contributions

A number of contributions are provided by this research. The major contributions are

- The first analytical and finite-element models of laminar jamming structures, which can accurately predict their static behavior over all major phases of deformation
- Scaling relations that show how critical performance metrics of laminar jamming structures scale with design parameters
- The first demonstration of shape-locking behavior using laminar jamming, as well as the first method for rapidly and efficiently turning joints on and off in soft robots
- The first lumped-parameter model of laminar jamming structures, which can approximate their dynamic behavior and provide intuition to designers
- The first demonstration of tuning impact responses of soft robots and aerial systems using laminar jamming

- The novel concept of jamming-based sandwich structures, which can overcome performance-per-weight limits of existing laminar jamming structures

Collectively, this thesis provides researchers with an analytical understanding of the statics and dynamics of laminar jamming structures; endows designers with scaling relations and computational tools for predicting the behavior of these structures and meeting performance requirements; and demonstrates how the structures can bridge the gap between soft robots and traditional rigid robots through increased mechanical versatility. Finally, this research conceives a new class of jamming structures that can push performance boundaries further.

Chapter 2

Transforming Static Behavior with Laminar Jamming

In this chapter, we rigorously investigate the mechanical behavior of laminar jamming structures and show that they can transform the static behavior of soft robotic systems. Specifically, we develop rigorous analytical and finite element models of laminar jamming, and we experimentally characterize jamming structures to show that the models are highly accurate. We then integrate jamming structures into soft robots to enable them to selectively exhibit the stiffness, damping, and kinematics of traditional rigid robots. The models allow jamming structures to be rapidly designed to meet arbitrary performance specifications, and the physical demonstrations illustrate how to construct systems that can behave like either soft robots or traditional rigid robots at will, such as continuum manipulators that can have joints appear and disappear.

2.1 Introduction

As partially described in Chapter 1, the structure of soft robots allows them to conform to complex shapes [15, 60], withstand crushing loads [19], dampen impacts, and interact safely with the body [27, 61]. In contrast, the structure of traditional rigid robots enables them

to perform tasks quickly, precisely, and with high resolution, as well as resist deformation, apply high forces, and oscillate with minimal decay. Researchers have attempted to make soft robots that can selectively behave like traditional rigid robots by integrating components with tunable stiffness and damping, such as low-melting-point materials [31, 33], shape-memory materials [62, 63], magnetorheological fluids [64], and granular structures [27, 44].

Nevertheless, most of these technologies cannot achieve a wide range of stiffness and damping values per unit weight (MR fluids, granular structures), have low resolution of stiffness and damping values (low-melting-point materials), transition between these values slowly (low-melting-point materials, shape-memory materials), and/or have poor resistance to bending moments (MR fluids, granular structures) [27, 28]. Furthermore, none of these technologies have yet enabled devices to rapidly transition between the continuous deformations typical of soft robots and the discrete, jointed deformations typical of traditional robots. Previous efforts towards this goal have been limited by slow transition times, high stiffnesses when no power is applied, and low efficiencies [31, 35].

In addition, although researchers have applied laminar jamming (Figure 2.1A-B) to areas as diverse as haptics [34, 50, 55], medical devices [51, 53], and soft actuators [58, 59], these studies have not yet provided analytical or computational models for laminar jamming beyond an initial deformation phase. As a result, the design of practical jamming structures is inevitably an arduous process. Moreover, researchers have not yet explored how laminar jamming can be used to transform bending kinematics.

In this chapter, we model laminar jamming in detail and demonstrate how the technology can bridge the gap between soft robots and traditional rigid robots. Specifically, we develop an analytical model that mathematically captures how two-layer jamming structures behave over all major phases of deformation. We then develop finite element models that extend these predictions to many-layer jamming structures, as well as describe how their stiffness and damping depend on critical design inputs (e.g., the vacuum pressure). These models are validated through rigorous experimental characterization. Together, the analytical and finite element models present researchers with the first means to rapidly and accurately design

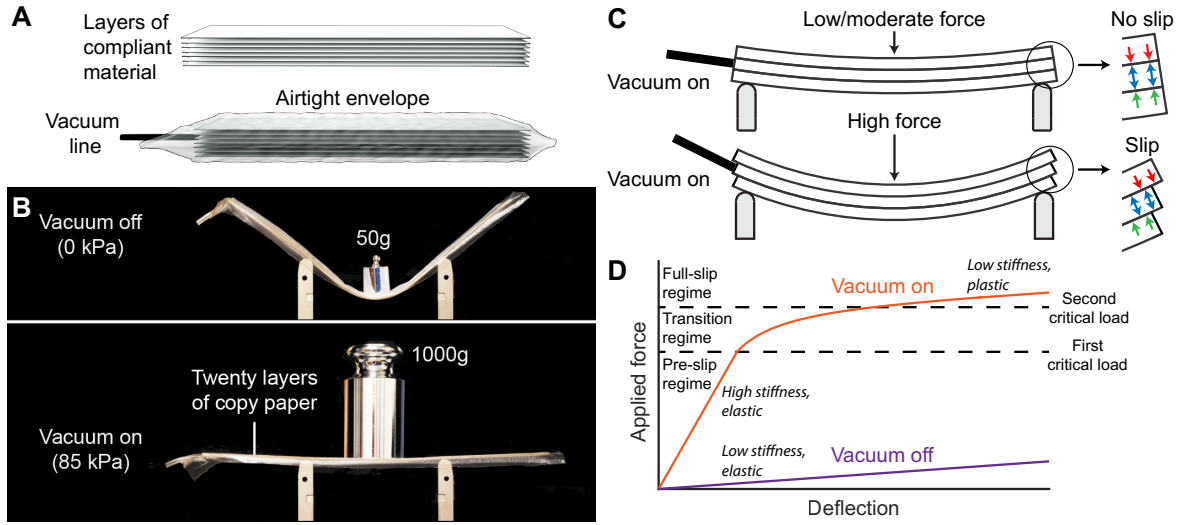


Figure 2.1: *Fundamental behavior of laminar jamming structures. A) Schematic of a jamming structure. B) When vacuum is off, the layers bend independently, and the structure has low bending stiffness. When vacuum is on, the layers bend as a cohesive unit, and the structure has high bending stiffness. C) However, when vacuum is on, the layers are cohesive only until a critical force. For higher forces, longitudinal shear stress is large enough to cause the layers to slip at certain points along their interfaces. D) Summary of mechanical behavior. When vacuum is off, the structure has low bending stiffness, which is proportional to the slope of the curves. When vacuum is on, the structure has three deformation regimes. In pre-slip, the bending stiffness is maximal and constant. After the first critical load, the structure enters the transition regime, in which the layers begin to slip. The bending stiffness decreases. After the second critical load, the structure enters full-slip, in which the layers have slipped at all possible points along their interfaces. The bending stiffness is minimal and constant. When slip occurs, energy is dissipated to friction between the layers, and the structure behaves plastically.*

jamming structures to meet arbitrary design requirements.

We then demonstrate the capabilities of laminar jamming structures by integrating them into real-world pneumatic and cable-driven soft robots. In the process, we achieve two novel functions that illustrate how these machines can reversibly emulate traditional rigid robots: 1) *shape-locking*, in which a compliant system can selectively manifest a stiff version of a desired shape and preserve it, even after powering off the actuators, and 2) *variable kinematics*, in which a compliant system can transition between continuous bending and discrete, jointed bending on command. The variable kinematics function is then used to build a two-fingered grasper that can perform pinch grasps on small objects, as well as wrap grasps on objects of eight times the diameter. These demonstrations prove the feasibility of using laminar jamming to build mechanically versatile machines and structures that exhibit both soft and traditional behavior.

2.2 Results

2.2.1 Analytical Modeling

As described earlier, when a vacuum is applied to a laminar jamming structure, the bending stiffness increases dramatically. Previous studies have shown that the stiffness increases by a factor of n^2 , where n is the number of layers in the structure; thus, applying a vacuum to a structure with just thirty-two layers can increase its stiffness by three orders of magnitude. However, the vacuumed jamming structure sustains this increased stiffness only for small loads, beyond which the stiffness declines [34, 51].

In our investigation, physical reasoning suggested that this behavior reflected three phases of deformation in a vacuumed jamming structure (Figure 2.1C-D): 1) In *pre-slip*, the layers are cohesive, and the stiffness of the structure is a factor of n^2 greater than the stiffness without vacuum. No energy is dissipated, and the damping (i.e., dissipated energy per unit deflection) is zero. As the structure is loaded, the longitudinal shear stress along the interfaces between layers begins to rise. 2) In the *transition regime*, the longitudinal shear stress along certain

regions of the interfaces equals the maximum possible shear stress, which is determined by the coefficient of friction and the pressure gradient. Layers begin to slip along those regions, and the stiffness of the structure decreases. Energy is dissipated to friction, and the damping increases. 3) In *full-slip*, all layers have slipped along the full length of their interfaces. The stiffness of the structure is minimal, and the damping is maximal.

To mathematically capture this behavior, we derived an analytical model that rigorously described the deformation and mechanical properties of jamming structures during these phases. Our model was based on Euler-Bernoulli beam theory; however, we extended the theory to describe how mechanical behavior was affected by vacuum pressure, friction at the interfaces between layers, and slip along the interfaces. Governing equations were derived using equilibrium and moment-stress relations, and general boundary conditions were formulated (Appendix A: Analytical Modeling: Governing Equations and Boundary Conditions). The boundary-value problem was then solved for a two-layer cantilevered jamming structure under a uniform distributed load (Appendix A: Analytical Modeling: Explicit Solution); this case was chosen to illustrate slip propagation (i.e., growth of the regions along which layers slip), which is exhibited by most jamming structures.

The model predicted the elastica (i.e., the shape), stiffness, dissipated energy, and damping of the jamming structure. The model also predicted the transition loads (i.e., the loads at which the jamming structure shifts from one deformation phase to the next), as well as the length of the region along which the layers slipped. Furthermore, it provided the functional dependence of all the preceding quantities on dimensions, material properties, the vacuum pressure, and the applied load (Appendix A: Analytical Modeling: Summary of Formulae). For example, the model showed that the full-slip damping force was given succinctly by μPbh , where μ is the coefficient of friction, P is the vacuum pressure, b is the width of a layer, and h is the height. Dimensionless forms of the equations in the model were derived as well (Appendix A: Analytical Modeling: Dimensionless Forms). The model was evaluated for an example structure (Figure 2.2), and the results were corroborated by two-layer finite element models (Appendix A: Finite Element Modeling: Two-Layer Jamming Structures).

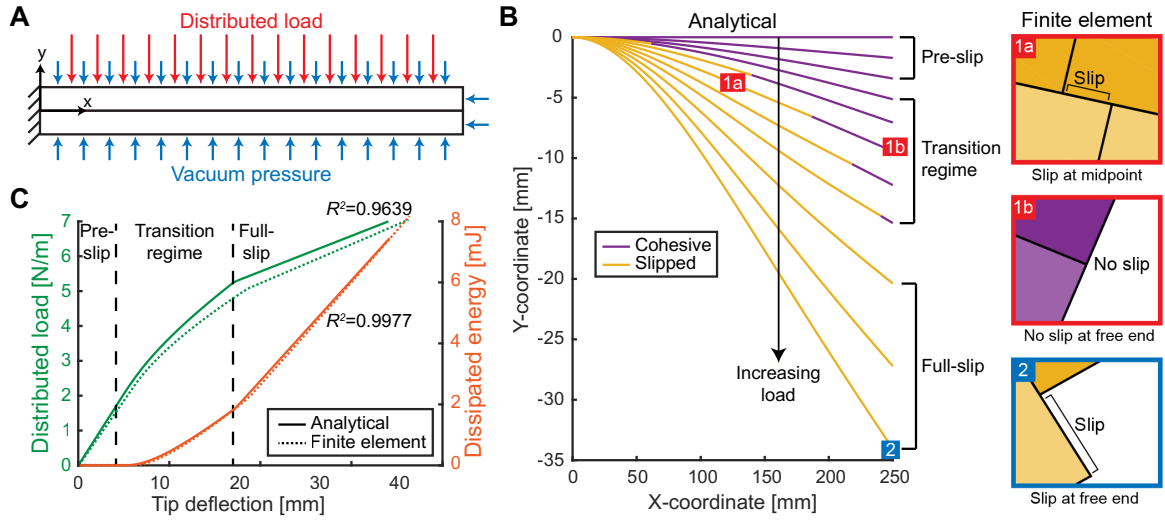


Figure 2.2: Analytical model of two-layer jamming structures. A) Schematic of example jamming structure. B) Elastica of jamming structure for increasing loads. The slipped region is highlighted at each load; because shear stress decreases along the x -direction, the slipped region initiates at the clamped end and grows toward the free end. Two-layer finite element models corroborated that slip occurred in analytically-predicted regions. C) Bending stiffness is proportional to the slope of the load-versus-deflection curve; as expected, the stiffness transitions from a minimal to a maximal value. Damping is proportional to the slope of the dissipated-energy-versus-deflection curve; damping transitions from zero to a maximal value. Finite element models closely corroborated analytically-predicted stiffness and damping values. (*Appendix A: Analytical Modeling: Case Study*)

2.2.2 Finite Element Modeling and Experimental Characterization

Although the analytical model rigorously predicted the mechanical behavior of two-layer jamming structures, designers may desire to build real-world jamming structures with additional layers to further adjust their properties. Our analytical model can be directly extended to describe many-layer jamming structures ([Appendix A: Analytical Modeling: Extending the Model](#)). However, the process is algebraically taxing, and numerical methods may be preferred.

To predict the mechanical behavior of many-layer jamming structures, we conducted finite element simulations. The jamming structures were modeled as 2D plane-strain structures with dimensions, material properties, boundary conditions, and loads equal to those of real-world jamming structures used later in experimental validation ([Appendix A: Finite Element Modeling: Stiffness and Damping of Many-Layer Jamming Structures](#)). Furthermore, simultaneous frictional contact was allowed to occur at all interfaces, and large-deformation analysis was enabled. No fitting parameters were used.

The results of the finite element simulations were used to quantify how critical design inputs affected major performance metrics of many-layer jamming structures. Specifically, the number of layers, vacuum pressure, and coefficient of friction of the layers were varied, and the stiffness and damping values of the jamming structures during pre-slip and full-slip were extracted. The polynomial relationship between each input and output was determined, and the resulting scaling relations were tabulated ([Appendix A: Finite Element Modeling: Functional Dependencies](#)). For example, full-slip damping was found to scale linearly with number of layers, vacuum pressure, and coefficient of friction.

To evaluate the accuracy of the finite element models, experimental characterization of many-layer jamming structures was conducted. Jamming structures were fabricated according to a multi-step process ([Appendix A: Experimental Characterization: Fabrication Process](#)), and the repeatability of the structures was assessed ([Appendix A: Experimental Characterization: Repeatability Analysis](#)). The jamming structures were highly repeatable from loading cycle to loading cycle and sample to sample. The many-layer jamming structures were then tested

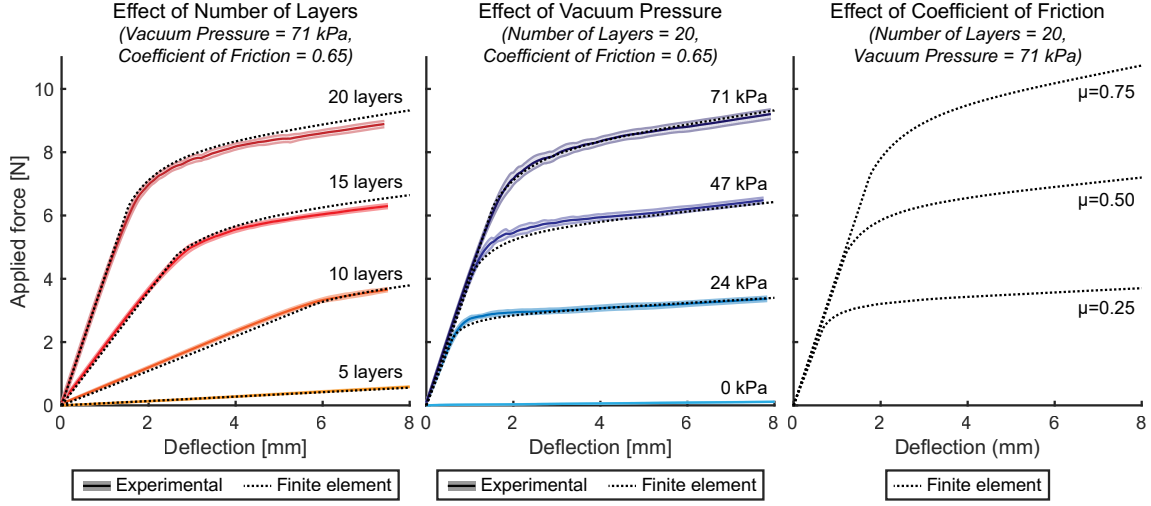


Figure 2.3: *Finite element predictions and experimental validation for many-layer jamming structures. Jamming structures were loaded in three-point bending. Each experimental curve in fact consists of a mean curve and shaded error bar that spans ± 1 standard deviation; the maximum deviation on any curve is 0.24N, indicating high repeatability. The minimum coefficient of determination (R^2) between finite element and experimental data is 0.9879, demonstrating exceptional predictive accuracy. (No experimental data is shown for coefficient of friction, as friction could not be precisely adjusted.) Hysteresis and damping predictions were experimentally evaluated as well (Figure A.6).*

in three-point bending for various numbers of layers and vacuum pressures (Appendix A: Experimental Characterization: Stiffness and Damping Characterization Process). Transverse force and maximum deflection was recorded, and finite element predictions were compared to experimental data (Figure 2.3). The finite element models predicted experimental results with exceptional accuracy.

2.2.3 Useful Functions

Shape-Locking

Two real-world capabilities of laminar jamming structures were demonstrated by integrating them into soft robots. First, the shape-locking function was demonstrated. A pneumatically powered soft bending actuator was fabricated (Appendix A: Functions and Applications: Shape-Locking), and a twenty-layer jamming structure was adhered to the ventral surface (i.e., the longitudinal surface closer to the center of curvature when the actuator was inflated). The

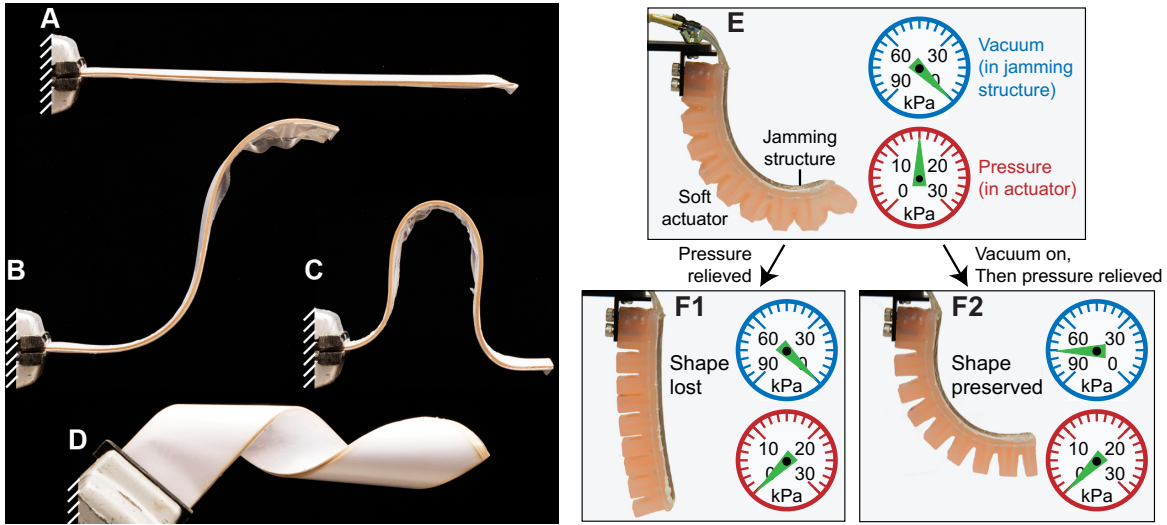


Figure 2.4: Demonstration of shape-locking function. A-D) A jamming structure consisting of twenty layers of copy paper was deformed into various shapes; vacuum was then applied, and the shape was preserved in all cases. E) A twenty-layer jamming structure was then adhered to a pneumatic soft actuator. The actuator was initially pressurized to 16kPa to achieve a desired bending angle. F1) In a first test, no vacuum was applied to the jamming structure, and the actuator was depressurized. The composite structure immediately returned to its undeformed state. F2) In a second test, a vacuum pressure of 85kPa was first applied to the jamming structure, and the actuator was then depressurized. Quantity R^2 between the arc of the ventral surface in E and the same arc in F2 was 0.9835. Thus, the system preserved its shape with high fidelity.

actuator was then pressurized. When the actuator was depressurized, the system naturally returned to its undeformed configuration; however, when a vacuum was applied to the jamming structure before the actuator was depressurized, the system preserved its shape with high fidelity (Figure 2.4).

Variable Kinematics

Next, the variable kinematics function was demonstrated. A robotic system was designed that consisted of three major parts: a silicone rubber substrate, a three-part jamming structure (i.e., three stacks of material, separated by narrow gaps), and a cable routed through the substrate to actuate bending (Figure 2.5A). Note that when the rubber substrate and the vacuumed state of the jamming structure are considered separately, their bending kinematics are entirely distinct. The substrate bends continuously along its length, whereas the vacuumed jamming structure bends discretely at its narrow gaps, which act as joints. When the substrate and

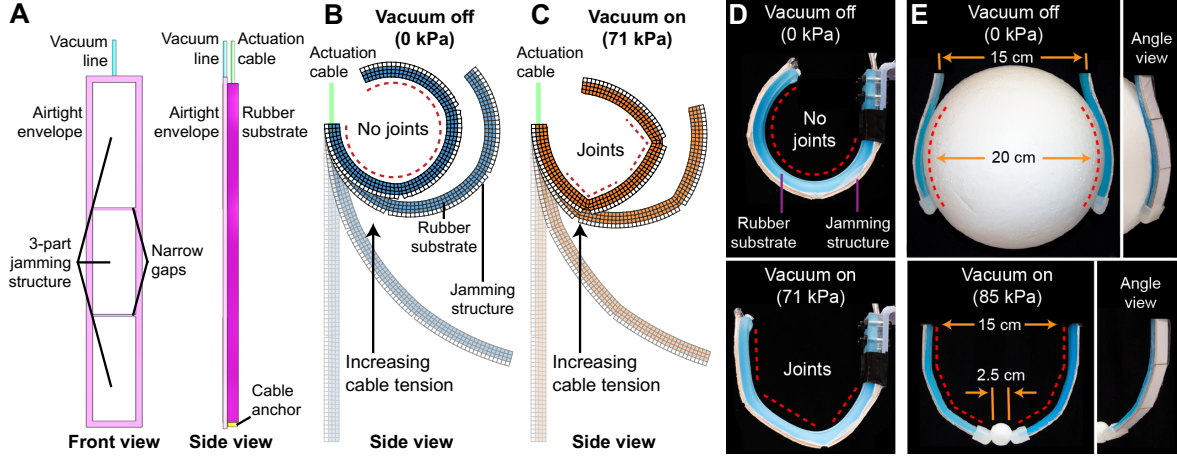


Figure 2.5: *Finite element modeling and experimental demonstration of variable kinematics function. A) Schematic of variable kinematics system. B-C) Finite element simulations of variable kinematics behavior at increasing cable loads. At the highest load, the ratio of maximum to mean curvature increased by a factor of 6.65 with vacuum on, quantitatively verifying the creation of joints. D) Experimental validation of variable kinematics system. E) A two-fingered grasper was constructed in which each finger consisted of a variable kinematics system with a rounded fingertip. When no vacuum was applied to the fingers, the grasper could conform to a large ball, hold it aloft, and resist perturbation, thus performing a stable wrap grasp. When vacuum was applied, the grasper could perform a stable pinch grasp on a ball of one-eighth the diameter.*

the jamming structure are adhered, the bending kinematics of the system may vary between these two extremes.

To enable the system to transition between continuous and discrete kinematics, the bending stiffnesses of the substrate and jamming structure were judiciously selected. The thickness of the substrate was chosen so that $k_{sub} = (k_{jam}^{nv} * k_{jam}^v)^{\frac{1}{2}}$, where k_{sub} is the bending stiffness of the substrate, k_{jam}^{nv} is the stiffness of the jamming structure without vacuum, and k_{jam}^v is the pre-slip stiffness of the jamming structure with vacuum. (In equivalent terms, k_{sub} was the geometric mean of the unjammed and jammed stiffnesses.) In addition, the number of layers in the jamming structure was chosen so that $k_{jam}^v \gg k_{jam}^{nv}$. Thus, when no vacuum was applied and the cable was pulled, the stiffness of the system would be dominated by k_{sub} , and the system would bend continuously. When vacuum was applied, the stiffness would be dominated by k_{jam}^v , and the system would bend discretely.

To evaluate this concept prior to prototyping, finite element simulations of the system were conducted ([Appendix A: Finite Element Modeling: Variable Kinematics](#)). The system

was modeled as a multi-part 2D plain-strain structure fixed at one end, and to approximate cable loading, a pure moment load was applied at the free end. The shape of the system was visualized, and the ratio of maximum to mean curvature ($\frac{\kappa_{max}}{\kappa_{mean}}$) was computed along the ventral arc as a measure of discreteness. When no vacuum was applied, the system deformed continuously, and $\frac{\kappa_{max}}{\kappa_{mean}}$ remained low. When vacuum was applied, the system deformed discretely, and $\frac{\kappa_{max}}{\kappa_{mean}}$ increased by a factor of 6.65 at high loads (Figure 2.5B-C).

Finally, a prototype of the system was fabricated (Appendix A: Functions and Applications: Variable Kinematics). The prototype deformed according to finite element predictions, and application of vacuum allowed it to select between continuous and discrete kinematics (Figure 2.5D).

2.2.4 Application

Two-Fingered Grasper

In robotic hands, compliant fingers that bend continuously can facilitate wrap grasps around large objects [17], whereas rigid fingers that bend discretely at joints can facilitate pinch grasps around smaller objects [11, 24]; it is challenging to design and fabricate hands capable of both. To accomplish the task, we built a two-fingered grasper in which each finger consisted of a cable-actuated variable kinematics system with a rounded fingertip. When no vacuum was applied and the cables were pulled, the fingers bent continuously, and the device could perform a stable wrap grasp on a ball of diameter 20 cm; when vacuum was applied first, the fingers bent discretely, and the device could perform a stable pinch grasp on a ball of one-eighth the diameter (Figure 2.5E).

To evaluate the stability of the grasps, multi-axis stiffness measurements were conducted and a perturbation test was performed (Appendix A: Functions and Applications: Two-Fingered Grasper). Stiffness measurements showed that the maximum bending stiffness of a finger increased by at least a factor of thirty-two when vacuum was applied. Simultaneously, the off-axis bending stiffness (i.e., the stiffness along the perpendicular bending axis) increased by a factor of 2.5, and the torsional stiffness increased by a factor of 2.7. Furthermore,

perturbation tests demonstrated that the force required to dislodge the small ball was at least a factor of eight greater when vacuum was applied and the discretely-bending fingers were used, compared to when no vacuum was applied and the continuously-bending fingers were used. As an additional feature, applying vacuum after the continuously-bending fingers performed a grasp increased their resistance to further deformation.

2.3 Discussion

2.3.1 Modeling

Earlier studies of laminar jamming exclusively predicted the stiffness of jamming structures during pre-slip, as well as the first transition load (i.e., the load at which the structures move from pre-slip to the transition regime) [34, 51, 65]. In contrast, our analytical model predicted the elastica, stiffness, energy dissipation, and damping of two-layer jamming structures during pre-slip, the transition regime, and full-slip, as well as determining both the first transition load and the second transition load (i.e., the load at which the structures move from the transition regime to full-slip). Our finite element models of many-layer jamming structures then extended the predictions of the analytical model to structures with arbitrary numbers of layers. Thus, the analytical and finite element models completely described the mechanical behavior of jamming structures over all three phases of deformation.

Together, the models provide designers with an accurate and efficient means to predict the mechanical behavior of arbitrary jamming structures. In particular, no models have existed for mechanical behavior in the transition regime or full-slip. To determine how a particular jamming structure will behave in these phases, designers have had to fabricate and characterize the structure. In our experience, this process requires hours of continuous labor per structure. In contrast, the analytical model can predict experimental behavior for a two-layer jamming structure immediately, and a finite element simulation can predict experimental behavior for a many-layer structure in less than one hour without supervision.

In addition, the functional dependencies of performance metrics on design inputs were

extracted for many-layer jamming structures. These relations provide researchers with a rapid means to meet arbitrary design requirements. For instance, if the full-slip stiffness of a jamming structure must be reduced by a factor of four (e.g., for an orthosis that softens at high loads for user safety), the relations show that the the number of layers or vacuum pressure can be reduced by a factor of four, or the coefficient of friction can be reduced by a factor of two (Table A.1). Likewise, if the full-slip damping of a jamming structure must be increased by a factor of four (e.g., for a field robot that dampens impacts to protect components), then the number of layers, vacuum pressure, or coefficient of friction can be increased by a factor of four. Note that vacuum pressure can be controlled on command with a vacuum regulator; thus, full-slip stiffness and damping can be adjusted in real-time. The finite element models can be used to derive functional dependencies between additional performance metrics and design inputs as desired. Note that although the models and experiments in this chapter investigated jamming structures with a maximum of twenty layers, the extracted functional dependencies are applicable to structures with arbitrary numbers of layers; however, as the number of layers increases, it may become challenging to physically achieve a homogeneous vacuum pressure throughout the structure.

2.3.2 Useful Functions

Previous studies applied laminar jamming to diverse applications. However, these studies almost exclusively used laminar jamming to control stiffness; furthermore, when the jamming structures were integrated with actuators, the structures controlled the stiffness of the system while the actuators were continuously powered. We expanded on these capabilities by demonstrating shape-locking and variable kinematics. The former enables soft robots to preserve their shape after the actuation input is removed, whereas the latter enables them to select between continuous and discrete bending.

Shape-locking illustrates one way in which laminar jamming structures can enable soft robots to reversibly emulate traditional rigid robots. Nearly all traditional robotic arms can navigate to an arbitrary location in their workspace and resist static loading. Furthermore,

some arms have brakes that allow them to resist loading after power is disconnected. Shape-locking endows soft robots with precisely this ability, as it enables them to achieve an arbitrary configuration, lock in place, and resist static loading, even after disconnecting the actuation input. Soft machines can thus save power by requiring no control effort to preserve their shape; furthermore, soft robots with high material strain (e.g., McKibben actuators) can be deflated after locking, mitigating the risk of catastrophic rupture. Shape-locking may also be achieved by combining soft actuators with other materials and structures capable of tunable stiffness and damping, such as recently-developed thermally-activated fibers and composites [66–68]; however, laminar jamming offers a promising combination of rapid actuation, human safety, and high range of bending stiffness between the deforming and shape-locked states.

Variable kinematics comprises a second way in which laminar jamming structures can link the behavior of soft robots and traditional rigid robots. Specifically, this function can allow soft robots to transform between a compliant state in which they can conform to arbitrary shapes, and a rigid, jointed state in which they can behave like a serial manipulator. As demonstrated in this chapter, variable kinematics can enhance the performance of robotic graspers. Moreover, this capability can be useful for any device where both conformability and rigidity are desired (e.g., in surgical devices that must traverse vasculature, but subsequently apply high forces).

More generally, variable kinematics facilitates the modeling, sensing, and control of soft robots. For traditional rigid robots, multi-rigid-body mechanics can describe forward and inverse kinematics; on the other hand, soft robots require the mathematical tools of continuum mechanics, which are generally far more complex. Furthermore, in traditional rigid robots, a small number of sensors can accurately estimate configuration; in soft robots, many sensors are required. Because modeling and sensing is more complex for soft robots, control is inherently more difficult [69, 70]. Variable kinematics allows soft robots to behave like multi-rigid-body systems, with rigid links connected by joints. Thus, they can be modeled and sensed like traditional rigid robots, greatly simplifying their control. (It is interesting to note that octopuses use variable kinematics to simplify control, creating joints along their tentacles to

facilitate fetching tasks [71].)

2.3.3 Limitations

Our modeling and demonstrations have three notable limitations, each of which can be resolved as described. First, in our finite element models of many-layer jamming structures, the execution time of the simulations scaled linearly with the number of layers; for models of jamming structures with exceptionally high numbers of layers, the time may become prohibitive. Nevertheless, as the numbers of layers increases in a jamming structure with a fixed total thickness, the structure may be accurately approximated as a single crystal with a single slip system. This structure can be simulated more simply than a multi-layer structure, reducing execution time ([Appendix A: Finite Element Modeling: Limiting Behavior](#)).

Second, in our shape-locking demonstration, our prototype still required a vacuum source to be connected after depressurization. Thus, the device would be challenging to operate in environments where supporting equipment is unavailable. This difficulty could be resolved by using a one-way valve to maintain vacuum after the vacuum input is disconnected.

Third, in our demonstrations, vacuum was used to actuate the jamming structures. As a result, the maximum pressure gradient acting on the jamming structures was limited to the absolute ambient pressure, which in turn reduced the maximum load that could be sustained by the structures before their stiffness declined. This limit may be overcome by using electrostatic actuation [65] or elastic actuation, in which the layers are reversibly compressed by an external elastic structure (e.g., a mesh envelope [58] or spring clips ([Appendix A: Additional Concepts: Spring-Based Jamming](#))).

2.4 Methods

The following is an abridged description of the methods used in this chapter. For complete detail, see [Appendix A](#).

2.4.1 Analytical Modeling

The axial strain fields in each layer of the jamming structure were approximated as a superposition of a field that varied linearly with the vertical coordinate and a field that was constant with the vertical coordinate. An interfacial displacement variable was defined. Moment-stress relations and static equilibrium were used to derive governing equations for sections of the structure with cohesive interfaces and sections with slipped interfaces. Boundary conditions were formulated for clamped and free boundaries, and continuity conditions were defined to couple cohesive and slipped interfaces. The boundary-value problem was then explicitly solved to determine the elastica of a cantilevered jamming structure with a uniform distributed load in the pre-slip regime, transition regime, and full-slip regime. During the transition regime, the location of the transition between cohesive and slipped interfaces was also determined. The results were then used to derive stiffness, energy dissipation, and damping in each regime, as well as critical loads between the regimes. Dimensionless parameters were defined to nondimensionalize all results.

2.4.2 Finite Element Modeling

All finite element models were constructed using finite element simulation software (ABAQUS 6.14r2, Dassault Systèmes, Villacoublay, France). In the finite element models of the two-layer and many-layer jamming structures, each layer was approximated as a 2D plane-strain structure. Pressure equal to vacuum pressure was applied to all outer surfaces, and loads were subsequently applied. Large-deformation analysis was enabled, and the interfaces between the layers were defined as contact surfaces with a penalty friction formulation. A uniform mesh was used consisting of square four-node bilinear plane-strain quadrilateral elements with reduced integration. Each layer was meshed with two elements across its thickness.

In the finite element models of the variable kinematics structures, the rubber substrate and each of the jamming structures was modeled as a *homogeneous* 2D plane-strain structure. To simulate the vacuum-on condition, the elastic modulus of the jamming structure was assigned to that of paper, and to simulate the vacuum-off condition, the modulus was reduced by a

factor of n^2 , with $n = 20$ to match experimental conditions. Cable actuation was approximated as a pure moment load. Large-deformation analysis was enabled. A uniform mesh was used consisting of square four-node bilinear plane-strain quadrilateral *hybrid* elements with reduced integration. The structure was meshed with four elements across its thickness.

2.4.3 Fabrication of Jamming Structures

The jamming structures were fabricated in five distinct steps. 1) Sheets of copy paper (HP Ultra White Multipurpose Copy Paper) were cut into strips on a laser cutter (VLS4.60, Universal Laser Systems, Inc., Scottsdale, AZ). 2) An acrylic frame enclosing the strips was cut on the laser cutter. The height of the acrylic frame was selected to be greater than the total thickness of the strips. The frame was used only for fabrication and did not comprise part of the jamming structure. 3) A sheet of thermoplastic polyurethane (TPU) (American Polyfilm, Inc., Branford, CT) was formed to the acrylic frame on a vacuum former (Formech 300XQ, Formech International Limited, Hertfordshire, UK). 4) The strips of paper and TPU tubing (Eldon James Corp., Denver, CO) were placed into the frame. The TPU sheet was folded over its contents, and the two sides of the sheet were sealed together on a heat press (Powerpress, Fancierstudio, Hayward, CA) at 100°C . Since the height of the acrylic frame was greater than the total thickness of the strips, the force of the heat press was concentrated on the frame; thus, only the region of the TPU sheet above the frame was sealed. The frame was then removed from the assembly. The remaining components comprised a jamming structure. 5) The end of the structure containing the TPU tubing was sandwiched between two conforming aluminum blocks. The blocks were heated to 171°C on the heat press, creating a circumferential seal around the tubing. The blocks were then removed from the assembly.

2.4.4 Experimental Characterization

Jamming structures were tested on a three-point bending fixture in a universal materials testing device (Instron 5566, Illinois Tool Works, Norwood, MA). The structures were placed on the fixture and connected to a manual vacuum regulator (EW-07061-30, Cole-Parmer,

Vernon Hills, IL) set to the desired pressure. The loading anvil of the testing device was lowered at a rate of 25 mm/min until reaching the desired maximum displacement. Force and displacement measurements were simultaneously recorded.

2.4.5 Functions and Applications

All molds were designed using CAD software (Solidworks 2015, Dassault Systèmes, Villacoublay, France) and 3D printed using a stereolithography-based printer (Objet30 Scholar, Stratasys, Ltd., Eden Prairie, MN). For the actuator used in shape-locking demonstrations, a two-part mold was designed, and the actuator was cast from shore 10A platinum-cure silicone rubber (Dragon Skin 10 Medium, Smooth-On, Inc., Macungie, PA). The actuator and jamming structure were bonded using silicone building sealant (Dow Corning 795, Dow Corning, Midland, MI).

For the substrate used in the variable kinematics demonstrations, a one-part mold was designed with an inserted rod to create a channel for an actuation cable. The substrate was cast from high-stiffness PDMS rubber (Sylgard 184, Dow Corning, Midland, MI). The substrate and three-part jamming structure were again bonded using silicone building sealant. The cable consisted of braided polyethylene (Hollow Spectra, BHP Tackle, Harrington Park, NJ) and was tensioned using a turnbuckle mechanism.

For the fingertips of the fingers in the two-fingered grasper, a two-part mold was designed, and the fingertip was cast from shore 00-10A silicone rubber (Ecoflex 00-10, Smooth-On, Inc., Macungie, PA). Multi-axis stiffness tests and perturbation tests were performed using a digital force gauge (Chatillon DFI10, AMETEK Sensors, Test & Calibration, Berwyn, PA) and custom-built fixtures.

2.5 Conclusions

This chapter has investigated the mechanical behavior of laminar jamming structures in detail and demonstrated that they can transform the static behavior of soft robotic systems. Specifically, we have derived an analytical model for two-layer jamming structures over all

major phases of deformation, constructed highly accurate finite element models of many-layer laminar jamming structures, and extracted functional dependencies of major performance metrics on critical design inputs. We have demonstrated two novel functions, shape-locking and variable kinematics, that illustrate how laminar jamming can reversibly endow soft robots with behavior typical of traditional rigid robots. We also built a simple grasper capable of both pinch grasps and wrap grasps, showing how laminar jamming can enhance the performance of real-world soft robotic systems. Collectively, the work in this chapter has elucidated the mechanics of laminar jamming, accelerated the design process of jamming structures, and provided a foundation for an overarching goal of this thesis—that is, creating mechanically versatile machines and structures that cannot simply be categorized as “soft” or “rigid.”

Chapter 3

Transforming Dynamic Behavior with Laminar Jamming

In this chapter, we continue to rigorously investigate the mechanical behavior of laminar jamming structures and now show that they can transform the *dynamic* behavior of soft robotic systems. Furthermore, they can do so in a form factor that overcomes the limitations of existing-variable damping mechanisms in size, cost, and convenience. Specifically, we combine analysis, simulation, and characterization to formulate a lumped-parameter model that captures the nonlinear mechanical behavior of jamming structures and can be used to rapidly simulate their dynamic response. We illustrate that by adjusting the vacuum pressure, the fundamental features of the dynamic response (i.e., frequency, amplitude, decay rate, and steady-state value) can be tuned on command. Finally, we integrate jamming structures into soft structures and aerial vehicles to demonstrate that laminar jamming can radically alter the impact response of soft and traditional rigid systems.

The physical experiments described in this chapter were conducted in collaboration with Alperen Değirmenci, who is also a co-author on the corresponding published article [72].

3.1 Introduction

The dynamic response of a robotic system is one of its most fundamental properties. It is defined as the transient and steady-state behavior of an output in response to a time-varying input (e.g., the oscillation in the position of a robotic arm after an impulse of force). When a robotic system interacts with the environment, actively controlling the dynamic response of the system can improve its safety, adaptivity, robustness, and energy efficiency [26].

As partially described in Chapter 1, the leading approach to controlling dynamic response is tuning mechanical impedance (i.e., stiffness and damping); thus, researchers have focused on developing variable-impedance mechanisms [26–28, 73]. However, existing mechanisms have notable limitations. In particular, variable-damping mechanisms are predominantly hydraulic (e.g., magneto/electrorheological fluids) or electromagnetic (e.g., Eddy currents); these systems are often large, heavy, expensive, and difficult to manufacture [26].

As discussed in Chapter 2 and the corresponding published article [49], laminar jamming structures (Figure 3.1) may also act as a variable-damping mechanism. When a jammed structure is initially deformed, its layers are cohesive, and its stiffness is maximal. However, when a critical load is applied, its layers begin to slip, and its stiffness decreases; moreover, energy is dissipated to friction between the layers. In this regime, the friction damping (i.e., energy dissipated per unit deflection) increases linearly with the external pressure gradient. In contrast to other variable-damping mechanisms, laminar jamming structures are thin, lightweight, low cost, and simple to fabricate. The controllable stiffness and damping of jamming structures may be combined to form a variable-impedance mechanism with a tunable dynamic response. Nevertheless, no studies have investigated these capabilities.

In this chapter, we demonstrate that laminar jamming can transform the dynamic response of robotic structures and systems. We combine analysis, simulation, and experiments to predict and measure the nonlinear static and dynamic behavior of jamming structures, and we determine that this complex behavior is captured by a lumped-parameter model that can be rapidly simulated. Furthermore, we show that by adjusting the pressure gradient applied to a jamming structure, the fundamental features of its dynamic response (i.e., frequency,

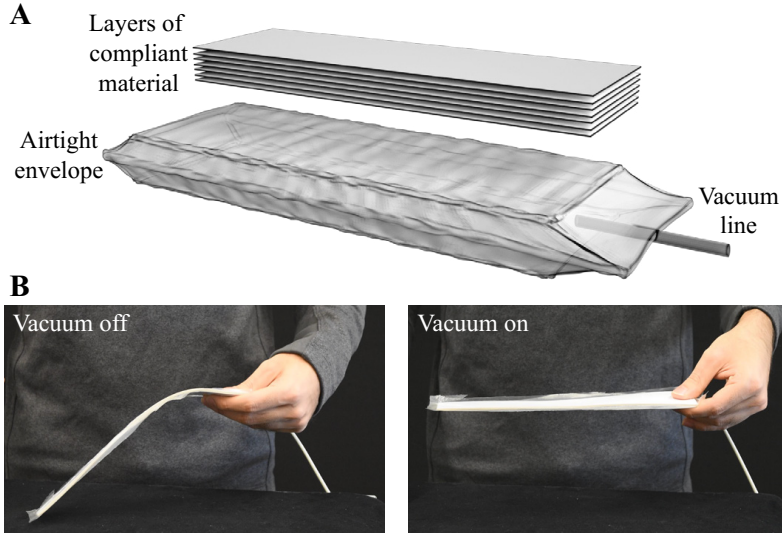


Figure 3.1: Vacuum-based implementation of laminar jamming. (A) Layers of compliant material are enclosed in an airtight envelope connected to a vacuum line. (B) When no vacuum is applied, the structure is compliant in bending. When vacuum is applied, the structure is stiff.

amplitude, and steady-state deformation) can be considerably altered. We then integrate laminar jamming structures into soft structures and unmanned aerial vehicles (UAVs) to illustrate that by adjusting the dynamic response of jamming structures, the impact response of both soft and traditional rigid robots can be transformed as well.

Thus, we demonstrate that laminar jamming is a useful variable-impedance mechanism that resolves several drawbacks of existing variable dampers. Furthermore, we provide designers with an analytical toolkit for building jamming structures to meet specific dynamic requirements.

3.2 Methods and Results

3.2.1 Development of Lumped-Parameter Models

In this section, we develop a method to rapidly predict how laminar jamming structures deform under static and dynamic loads. To do so, we measure quasi-static force-deflection curves of jamming structures and predict them using finite element simulations. However, these simulations can be difficult to generalize and require days to complete. Thus, we

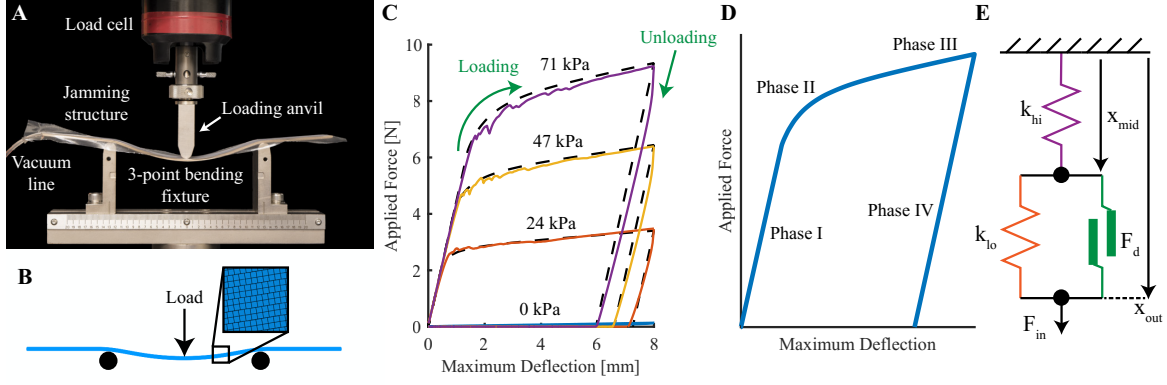


Figure 3.2: *Quasi-static behavior of laminar jamming structures. (A) Experimental characterization setup. (B) Corresponding finite element model. Inset shows slip between adjacent layers. (C) Experimental (colored) and finite element (dashed) force-deflection curves over one loading cycle at four different vacuum pressures. (D) Fundamental phases of deformation. (E) Schematic of quasi-static lumped-parameter model.*

subsequently formulate a quasi-static lumped-parameter model that provides intuition and can execute in seconds. We calibrate the model and show that it can predict experimental force-deflection curves. Finally, we formulate a dynamic lumped-parameter model and predict dynamic responses, which are validated in the next section.

Quasi-Static Experimental Characterization

Prior to experimental characterization, a jamming structure was fabricated. The structure consisted of twenty 250 mm x 50 mm layers of copy paper enclosed in an airtight envelope made of 0.076 mm-thick thermoplastic elastomer film (Stretchlon 200, Fibre Glast Developments Corp., Brookville, OH). The structure was then characterized on a materials testing machine (Instron 5566, Illinois Tool Works, Norwood, MA) in three-point bending (Figure 3.2A). The structure was centered on a bending fixture with the supporting anvils placed 130 mm apart and connected to a vacuum regulator set to the desired vacuum pressure (defined as the pressure inside the envelope below ambient pressure). A loading anvil attached to a 100 N load cell was lowered until contacting the structure. Force and displacement data were then recorded as the anvil was lowered by 8 mm and returned to its original position at $25 \frac{\text{mm}}{\text{s}}$.

The laminar jamming structure was tested at four different vacuum pressures, with three

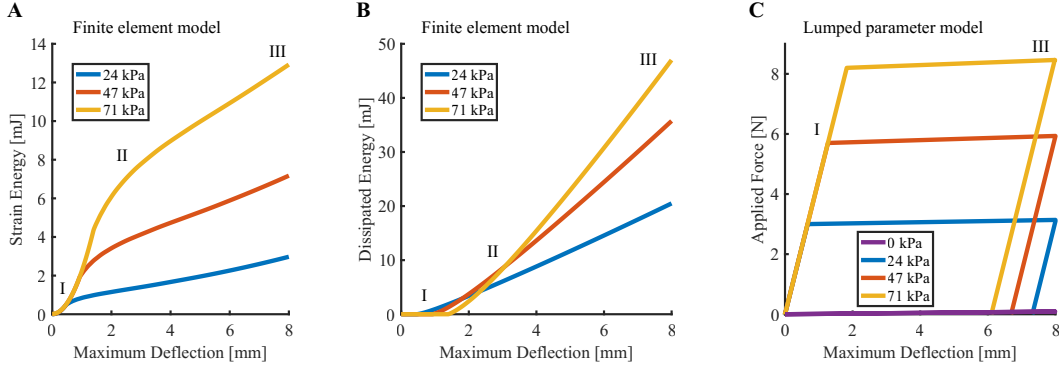


Figure 3.3: Calibration and simulation of quasi-static lumped-parameter model. (A) Finite element results for strain energy versus maximum deflection. (B) Finite element results for dissipated energy versus maximum deflection. (C) Quasi-static simulations of lumped-parameter model in three-point bending.

trials per pressure. For each vacuum pressure, mean force-displacement curves were computed. The results are shown in Figure 3.2C.

Quasi-Static Finite Element Modeling

Laminar jamming structures were modeled using finite element software (ABAQUS v6.14r2, Dassault Systemes, Villacoublay, France) according to a procedure first described in recent work [49]. Each layer was modeled as a 2D plane-strain structure with dimensions equal to their experimental dimensions. The elastic modulus E and static coefficient of friction μ were equal to measured values ($E = 6$ GPa, $\mu = 0.65$), and the Poisson's ratio ν was equal to the literature value ($\nu = 0.156$) [74]. At the interfaces between layers, frictional contact was prescribed. Large-deformation analysis was enabled.

The structure was constrained in three-point bending, and pressure (equal to the vacuum pressure) was applied to the outer edges. A linearly increasing force was applied to the center of the top layer over 200 equal increments until a deflection of 8 mm (Figure 3.2B). The load was then linearly decreased to zero over 200 equal increments. Simulation results are shown in Figure 3.2C. Note that no simulation results are provided for a vacuum pressure of 0 kPa, as the model was unstable without pressure.

Finite element results agreed closely with experimental results, demonstrating that the

behavior of jamming structures can be accurately predicted using finite element simulations. Furthermore, the results showed that the force-deflection curves at all nonzero vacuum pressures exhibited classical hysteresis loops. The energy dissipated per unit cycle (i.e., area under the hysteresis curves) and friction damping (i.e., energy dissipated per unit deflection) scaled linearly with vacuum pressure, indicating that damping can be controlled by simply adjusting vacuum pressure.

Each hysteresis loop consisted of four distinct phases (Figure 3.2D). In Phase I, the bending stiffness (proportional to the slope of the curve) was maximal and constant. In Phase II, the stiffness gradually decreased. In Phase III, the stiffness was minimal and approximately constant. Finally, in Phase IV, the structure was unloaded, and the slope of the curve matched that of Phase I. From the finite element simulations, it was found that in Phases I and IV, no slip occurred at the interfaces between the layers, and no energy was dissipated. In Phase II, slip occurred at increasingly long sections of the interfaces, and in Phase III, slip occurred at all possible sections of the interfaces; in both Phase II and Phase III, energy was dissipated to friction.

Quasi-Static Lumped-Parameter Model

The lumped-parameter model consisted of a stiff spring with stiffness k_{hi} in series with a parallel unit, which itself consisted of a compliant spring with stiffness k_{lo} (where $k_{lo} \ll k_{hi}$) and friction damper with damping force F_d (Figure 3.2E). The springs had an equilibrium length of zero. Force F_{in} modeled the transverse force applied to the jamming structure, and deflection x_{out} modeled the maximum deflection.

When F_{in} is small, the damper is rigid, and the compliant spring cannot deform. Thus, k_{hi} governs the stiffness of the system. This phase of deformation corresponds to Phase I. As F_{in} increases, the force on the damper exceeds F_d . The damper is no longer rigid, and the compliant spring can deform. Because $k_{lo} \ll k_{hi}$, quantity k_{lo} governs the stiffness of the system. This phase corresponds to Phase III. Finally, when F_{in} is decreased, the damper is once again rigid, and k_{hi} governs the stiffness of the system. This phase corresponds to Phase

IV. Note that Phase II is not modeled.

Before the lumped-parameter model could be simulated, its coefficients needed to be calibrated. Finite element results were chosen as reference data; however, experimental data could have been used as well.

Energetic equivalence was prescribed between the finite element model and the lumped-parameter model. Strain energy E_s and dissipated energy E_d were extracted from the finite element model over Phases I-III and plotted versus deflection (Figure 3.3A-B). For the lumped-parameter model, analytical expressions for dissipated energy and strain energy can be derived. For positive F_{in} , these energies are

$$E_s = \frac{1}{2}(k_{hi}x_{mid}^2 + k_{lo}(x_{out} - x_{mid})^2), \quad (3.1)$$

$$E_d = F_d(x_{out} - x_{mid}) . \quad (3.2)$$

During Phase I, $x_{out} = x_{mid}$, as the springs have an initial length of zero and only the stiff spring can deform. Furthermore, during Phase III, x_{mid} is approximately constant, as $k_{lo} \ll k_{hi}$ and the additional deformation of the stiff spring is negligible. Thus, during Phase I,

$$k_{hi} = \frac{\partial^2 E_s}{\partial x_{out}^2} . \quad (3.3)$$

During Phase III,

$$k_{lo} = \frac{\partial^2 E_s}{\partial x_{out}^2}, \quad (3.4)$$

$$F_d = \frac{\partial E_d}{\partial x_{out}} . \quad (3.5)$$

Given energetic equivalence, these formulae were applied to finite element results to determine k_{hi} , k_{lo} , and F_d at each vacuum pressure. The results are aggregated in Table 3.1. Note that k_{hi} was identical for all nonzero pressures. At 0 kPa, no friction is present; thus, $k_{hi} = F_d = 0$. Since no finite element simulations were conducted at 0 kPa, k_{lo} was calculated using the previously described analytical result that the bending stiffness of a jamming

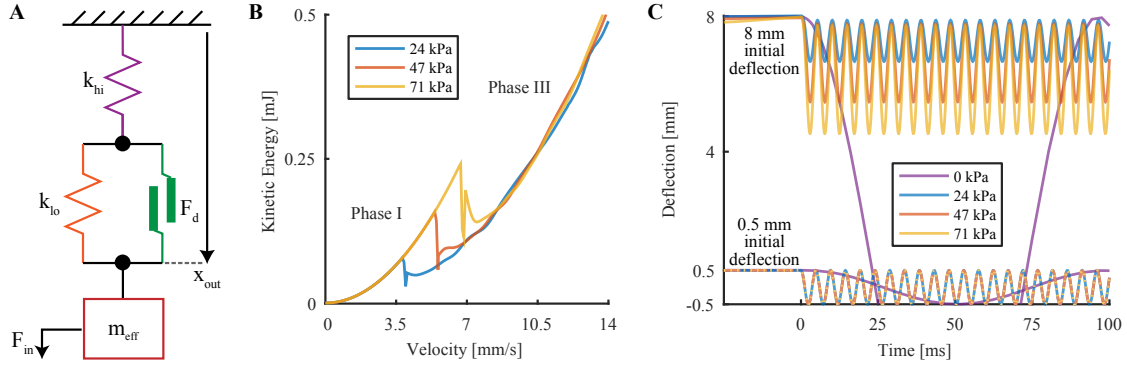


Figure 3.4: Calibration and simulation of dynamic lumped-parameter model. (A) Schematic of dynamic lumped-parameter model. (B) Finite element results for kinetic energy versus velocity at the point of maximum deflection. (C) Simulated step responses of lumped-parameter model.

structure without vacuum is equal to its initial stiffness with vacuum, divided by n^2 [34, 49, 65].

The lumped-parameter model was then simulated using dynamic simulation software (SimScape 2016b, The MathWorks, Inc., Natick, MA). The simulations executed in seconds on a laptop computer. The results are shown in Figure 3.3C. As desired, lumped-parameter results closely matched experimental and finite element data (Figure 3.2C). Of course, Phase II was not replicated; however, designers can still use the model to rapidly predict stiffness and energy dissipation for both small and large loads.

Dynamic Lumped-Parameter Model

Although the preceding lumped-parameter model accurately predicted the quasi-static behavior of jamming structures, it could not simulate the dynamic response (e.g., step response). Since the dominant damping phenomenon in laminar jamming structures is dry friction, which is velocity-independent, it was hypothesized that to simulate dynamics, only an additional effective mass was needed (Figure 3.4A).

To calibrate the magnitude m_{eff} of the effective mass, a dynamic-implicit finite element simulation was executed. The simulation had identical parameters to the quasi-static simulation; in addition, the mass density of the layers was equal to the experimental value (i.e., $7.75e3 \frac{kg}{m^3}$).

Table 3.1: *Coefficients of lumped-parameter model (3-point bending)*

Pressure [kPa]	Coefficient			
	$k_{hi}[\frac{N}{mm}]$	$k_{lo}[\frac{N}{mm}]$	$F_d[N]$	$m_{eff}(g)$
0	N/A	0.0112	0	5.4
24	4.48	0.0192	2.99	5.4
47	4.46	0.0345	5.71	5.4
71	4.46	0.0432	8.24	5.4

Energetic equivalence was again prescribed between the finite element model and the lumped-parameter model. Kinetic energy was extracted from the finite element results, plotted versus velocity at the point of maximum deflection, and low-pass filtered with cutoff frequency 3 Hz (Figure 3.4B). As before, no simulation results are provided for 0 kPa, as the model was unstable without pressure. The kinetic energy exhibited an anticipated transient dropoff and numerical noise when Phase I ended and energy dissipation commenced. The kinetic energy of the lumped-parameter model is

$$E_k = \frac{1}{2}m_{eff}v_{out}^2 \quad (3.6)$$

where v_{out} is velocity at $x = x_{out}$. Thus,

$$m_{eff} = \frac{\partial^2 E_k}{\partial v_{out}^2}. \quad (3.7)$$

Quantity m_{eff} was identical during Phase I and Phase III and constant at all pressures (Table 3.1). (Since no finite element simulations were conducted at 0 kPa, m_{eff} was assigned to the value at the nonzero pressures.) This result was expected, as the jamming structure experienced small displacements; thus, the geometry (and in turn, the mass distribution) of the structure does not change significantly between phases or pressures. Note that m_{eff} is nearly identical to the value calculated from the well-known formula for the effective mass of a simply-supported beam (i.e., $m_{eff} = 0.5m$) [75]. Thus, in subsequent simulations, effective-mass formulae can be used.

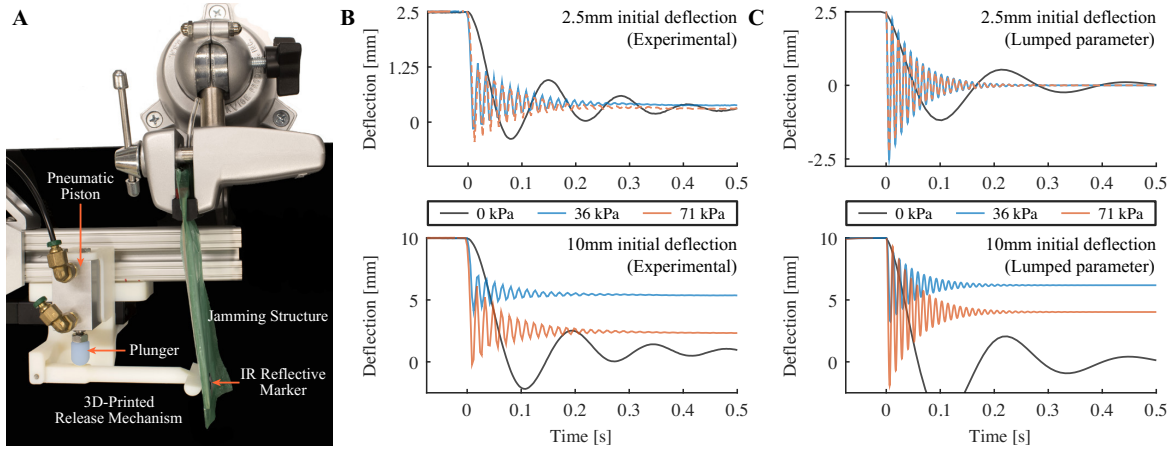


Figure 3.5: *Characterization and simulation of dynamic response of jamming structures. (A) Experimental setup to measure step response of jamming structure. (B) Experimental step responses at small and large initial deflections, each at multiple vacuum pressures. (C) Corresponding lumped-parameter step responses.*

A step response was then simulated. The model was initially displaced to deflections of 0.5 mm and 8 mm, and F_{in} was then released. Figure 3.4C shows the subsequent time responses for x_{out} at each vacuum pressure.

For the 0.5 mm initial deflection, the loads were not large enough at any pressure to deform the damper. As expected, the nonzero-pressure conditions oscillated with equal frequencies, as k_{hi} was effectively identical. However, the zero-pressure condition oscillated with a frequency that was a factor of n lower, as its stiffness was n^2 smaller. The amplitudes in the nonzero- and zero-pressure conditions were all identical, as no energy was dissipated to friction.

For the 8 mm initial deflection, the loads were large enough at all pressures to deform the damper. Identical to the 0.5 mm cases, the nonzero pressure conditions oscillated with equal frequencies, and the zero-pressure condition oscillated with a frequency that was a factor of n lower; furthermore, the zero-pressure condition maintained its initial amplitude, as no energy was dissipated. However, for the nonzero-pressure conditions, as pressure increased, the oscillations had larger amplitudes and a mean value that was closer to initial equilibrium (i.e., 0 mm). These results can be expected; as pressure increases, the ratio of strain energy to dissipated energy after F_{in} is released also increases.

The preceding results indicate that the variable-stiffness behavior of a jamming structure

may enable the oscillation frequency to be markedly altered by simply turning vacuum on and off. Furthermore, for large initial deflections, the variable-damping behavior of the structure may allow the amplitude of oscillation and steady-state deformation to be tuned on command by adjusting vacuum pressure.

3.2.2 Evaluation of Dynamic Lumped-Parameter Model

In the previous section, we used quasi-static experiments, simulation, and analysis to propose a dynamic lumped-parameter model. In this section, we experimentally evaluate the lumped-parameter model and show that it indeed predicts dynamic responses; thus, designers can use the models to rapidly predict dynamic behavior.

Although quasi-static force-deflection behavior was analyzed in three-point bending, we now examine dynamic responses in cantilever bending. This loading condition is more common in real-world dynamic robots (e.g., manipulators); furthermore, the investigation shows that the analytical methods proposed are agnostic to loading conditions and may be used by designers in diverse physical scenarios.

Dynamic Experimental Characterization

The dynamic response of laminar jamming structures was characterized on a custom experimental setup (Figure 3.5A). A twenty-layer 125 mm x 50 mm jamming structure was fabricated, and a circular fiducial marker was cut from infrared (IR)-reflective fabric (RF-HW25400, Nanning V-Can Business Co., Ltd., Nanning, China) and adhered to the jamming envelope. The position of the marker was measured using an optical tracking system (FusionTrack 500, Atracsys, Switzerland) with an accuracy of 90 μm and sampling rate of 335 Hz.

At the beginning of each test, the jamming structure was connected to a vacuum regulator set to a desired vacuum pressure, and the structure was clamped horizontally. When a pneumatic solenoid was actuated, a plunger pushed a release mechanism that rapidly rotated and released the tip of the jamming structure, implementing a step force input.

Two jamming samples were tested with initial deflections of 2.5 mm and 10 mm at vacuum

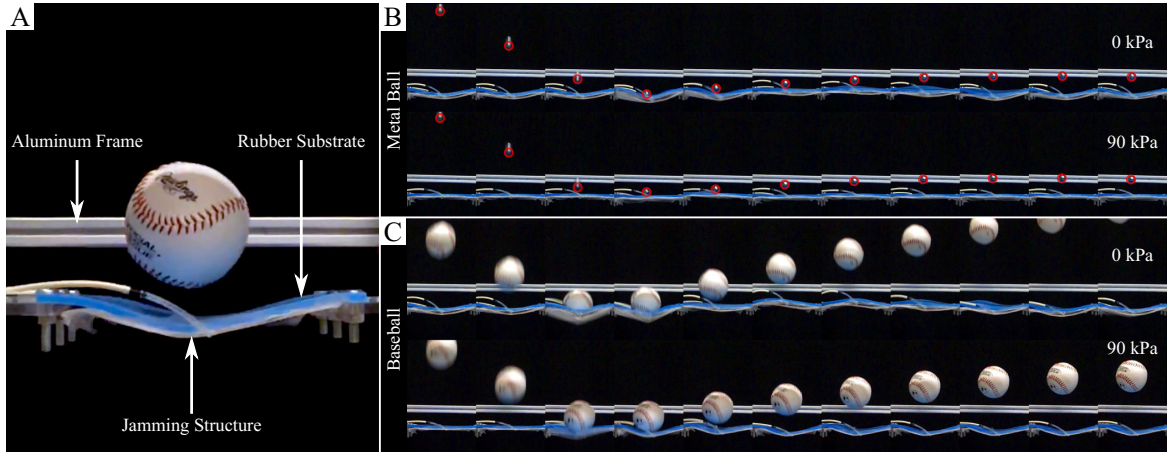


Figure 3.6: *Tuning the impact response of soft structures using laminar jamming. (A) Experimental setup prior to collision. (B) Representative time series of collision with a metal ball without and with vacuum. Ball is outlined in red. (C) Representative time series of collision with a baseball.*

pressures of 0 kPa, 36 kPa, and 71 kPa. Mean time responses are shown in Figure 3.5B.

Dynamic Lumped-Parameter Model

Coefficients of the lumped-parameter model for cantilever bending were extracted using the process described earlier. Energetic equivalence was again prescribed, and (3.3)-(3.5) were used to compute the coefficients for the springs and dampers. As validated earlier, effective-mass formulae were used to determine m_{eff} (for a cantilever beam, $m_{eff} = \frac{33}{140}m$). The coefficients are provided in Table 3.2. The resulting time responses are also shown in Figure 3.5C; note that to model experimentally-observed air drag, the model responses were multiplied by exponential decay functions with empirically determined time constants of 0.14 s for the zero-pressure conditions and 0.05 s for the nonzero-pressure conditions.

The lumped-parameter model exhibited the same fundamental trends as the experimental data. In both the experimental and lumped-parameter results, for both small and large deflections, the oscillation frequencies of the nonzero-pressure conditions were approximately identical, and the frequencies of the zero-pressure conditions were approximately a factor of n lower. (Experimental frequency reductions were slightly less than n due to imperfect clamping of the jamming structures.) In both the experimental and lumped-parameter results

Table 3.2: *Coefficients of lumped-parameter model (Cantilever)*

Pressure [kPa]	Coefficient			
	$k_{hi}[\frac{N}{mm}]$	$k_{lo}[\frac{N}{mm}]$	$F_d[N]$	$m_{eff}(g)$
0	N/A	0.00153	0	2.0
36	0.614	0.0040	2.10	2.0
71	0.614	0.0086	3.71	2.0

for the large initial deflection, oscillation amplitudes increased with vacuum pressure, and steady-state deformations were closer to initial equilibrium (i.e., 0 mm). These trends are identical to those observed for the model in three-point bending and have the same physical basis.

The lumped-parameter results were also numerically accurate. For both small and large deflections at 0 kPa and 36 kPa, as well as small deflections at 71 kPa, the steady-state deformation value for the lumped-parameter model was within 0.5 mm of the experimental value. However, for large deflections at 71 kPa, the steady-state value for the model (≈ 4.5 mm) was greater than the experimental value (≈ 2.5 mm). This discrepancy likely arises from neglecting Phase II in the model, which consequently underestimates the elastic energy stored in the structure prior to release.

To provide additional physical insight, dynamic finite-element simulations of the post-release oscillations were executed, and dissipated energy was extracted. It was found that all energy dissipated to friction was dissipated immediately. In terms of the lumped-parameter model, the friction damper deformed immediately after release; however, once the damper stopped deforming, it never deformed again.

The preceding results demonstrate that lumped-parameter models based on quasi-static cyclic loading tests can accurately predict the dynamic response (e.g., step response) of jamming structures. Furthermore, the experimental and lumped-parameter results offer further evidence that vacuum pressure can dramatically alter dynamic response. By applying vacuum pressure, the variable-stiffness properties of jamming structures can increase the oscillation

frequency by approximately a factor of n . Furthermore, by increasing vacuum pressure, the variable-damping properties of the structures can increase the oscillation amplitude and drive steady-state deformation closer to initial equilibrium.

3.2.3 Controlling Impacts with Laminar Jamming

In previous sections, we showed that by adjusting the vacuum pressure, the dynamic response of jamming structures can be altered on command. In this section, we demonstrate that by integrating jamming structures into real-world robotic structures and systems, the impact responses of such systems can be transformed as well.

Tuning the Impact Response of Soft Structures

A twenty-layer 150 mm x 150 mm jamming structure was fabricated. To represent a typical soft-robotic structure, a 5 mm-thick substrate of identical area was cast from Shore 50A silicone rubber (Smooth-Sil 950, Smooth-On, Macungie, PA), a material used in soft actuators. The jamming structure was then adhered to the rubber. The composite structure (i.e., the substrate and jamming structure) was bolted on two sides to an aluminum frame (Figure 3.6A); slack was introduced to reduce the effect of membrane forces on initial stiffness.

A 25 mm-diameter steel ball (28 g) and a 72 mm-diameter baseball (158 g) were dropped onto the composite structure from a height of 1.5 m with and without vacuum applied to the jamming structure. An alignment fixture was constructed to drop the objects with high positional repeatability. The collisions were filmed at 240 fps using a high-speed video camera. Representative time series are shown in Figure 3.6B-C.

Figure 3.6B illustrates that the metal ball rebounded to similar heights with and without vacuum. These results imply that the impact forces were not large enough to drive the jamming structure from Phase I to Phase II, and that energy was not dissipated in the jamming structure; this conclusion is further supported by the images of the vacuum condition, which show negligible deformation of the composite structure.

On the other hand, Figure 3.6C illustrates dramatically different dynamic responses for

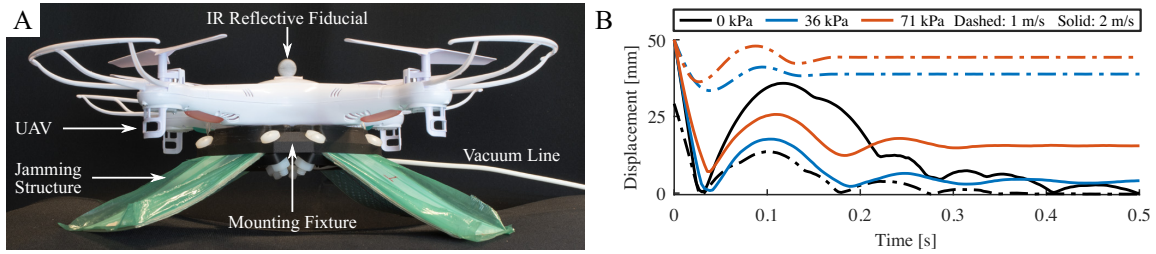


Figure 3.7: *Tuning the impact response of a UAV using laminar jamming. (A) UAV with landing gear consisting of four jamming structures. (B) Time response of UAV chassis displacement at two different landing velocities and three different vacuum pressures. Displacement is relative to bottom-out position (i.e., bottom-out = 0 mm). Time is relative to touchdown (i.e., initial contact = 0 s).*

the baseball. The baseball without vacuum repeatably rebounded to a minimum of three times the height of the baseball with vacuum. These results imply that impact forces were indeed enough to drive the jamming structure into Phase II, and that significant energy was dissipated in the jamming structure; this conclusion is validated by the images of the composite structure after collision, which show transient oscillations without vacuum, but a large permanent deformation with vacuum.

The preceding results demonstrate that jamming structures can effectively tune the impact responses of soft-robotic structures. Without vacuum, the energy of an incoming projectile and the original equilibrium deformation of the target can both be maximally preserved; with vacuum, the energy of the incoming projectile can be sharply dissipated.

Tuning the Impact Response of Traditional Rigid Robots

Four twenty-layer 125 mm x 50 mm jamming structures were fabricated. A 3D-printed fixture was designed that cantilevered the jamming structures at 30° from the bottom of a UAV (Syma X5C Quadcopter, Guangdong Syma Model Aircraft Indl Co Ltd, Shantou, China), constituting landing gear (Figure 3.7A). As in the dynamic experimental characterization, an IR-reflective fiducial marker was mounted on the UAV and tracked using an IR camera at 335 Hz.

The tests were designed to simulate slow and fast landings of a UAV. It was hypothesized that for a given landing velocity, there existed an ideal vacuum pressure for the jamming

structures that would simultaneously minimize peak forces on the UAV while also preventing its chassis from bottoming out (i.e., striking the ground).

During each test, the vacuum pressure on all four jamming structures was set to a desired level. The system was then maneuvered to desired landing velocities by adjusting drop height and propeller speed. The resulting collisions were filmed at 60 fps and are illustrated in Figure 3.7B.

On immediate inspection, the results once again demonstrate that by adjusting vacuum pressure on laminar jamming structures, the dynamic response of a robotic system can be transformed. For a given velocity, different pressures showed markedly distinct oscillation amplitudes, frequencies, and decay rates. Furthermore, the results supported our hypothesis; at a given landing velocity, an ideal pressure for the jamming structures did exist. Specifically, it was the minimum pressure that still prevented the chassis from bottoming out. At a landing velocity of 1 m/s, the 0 kPa condition bottomed out and exhibited the highest peak forces (approximated by the second derivative of the displacement-versus-time curve). Among the remaining pressure conditions, the 36 kPa condition was ideal, as it exhibited lower peak forces and a higher decay rate. Given that the initial stiffnesses of the jamming structures in the nonzero-pressure conditions are identical, the higher initial oscillation amplitude in the 36 kPa condition indicates that the structures entered Phase II and dissipated kinetic energy. At a landing velocity of 2 m/s, both the 0 kPa and the 36 kPa conditions bottomed out; however, the 71 kPa condition did not and was thus ideal.

The preceding results demonstrate that jamming structures can also be integrated into traditional rigid robotic systems to rapidly tune impact responses. Furthermore, given their light weight, high damping force range, and effectively infinite damping resolution, jamming structures may constitute a compelling mechanism for active UAV landing gear.

3.3 Discussion

The present work offers several contributions. First, it demonstrates that laminar jamming structures constitute an effective variable-impedance mechanism that overcomes several

limitations of existing mechanisms, particularly variable dampers. As described earlier, by applying vacuum pressure, the stiffness of a jamming structure can be increased by a factor of n^2 , where n is the number of layers; furthermore, for large forces (i.e., in Phase III), energy dissipation and friction damping scale linearly with the vacuum pressure. Through experiments and simulations, it was shown that the variable-stiffness properties of jamming structures allow oscillation frequencies to be adjusted, whereas the variable-damping properties enable oscillation amplitudes, decay rates, and steady-state deformations to be tuned on command. Furthermore, these functions are realized in a form that is thin, lightweight, low cost, and easy to manufacture, ameliorating the drawbacks of existing mechanisms.

Second, the chapter illustrates that laminar jamming can effectively tune the dynamic responses of real-world robotic structures and systems. Through experimental demonstrations, it was shown that the impact response of soft structures (i.e., soft substrates with integrated jamming structures) can be tuned to conserve or dissipate energy as desired. For an incoming projectile with high kinetic energy, relieving vacuum pressure from the jamming structures allows the energy of the projectile to be maximally preserved and the composite structure to return to equilibrium; on the other hand, applying vacuum pressure to the jamming structures rapidly dissipates the energy of the projectile and preserves the composite structure in a deformed state.

In addition, it was shown that the impact response of aerial robots can be tuned by constructing landing gear consisting of laminar jamming structures. By applying an appropriate vacuum pressure to the jamming structures (ideally, the lowest possible pressure that still prevents bottoming out), peak forces can be minimized, decay can be accelerated, and shock loads can be mitigated. Thus, jamming-based landing gear has potential to meet the design requirements of aerial vehicles in the robotics literature (e.g., [76, 77]). In addition, since vacuum pressure can rapidly adjust the mechanical properties of jamming structures, the ideal vacuum pressure can be selected right before collision, eliminating the requirement to land at a predetermined speed.

As a third contribution, the present work provides designers with an analytical toolkit

for rapidly predicting the dynamic response of laminar jamming structures. In general, jamming structures exhibit highly nonlinear, hysteretic behavior that can be challenging to anticipate; this chapter formulated and experimentally validated a dynamic lumped-parameter model that captures the essential features of the dynamic response (i.e., oscillation frequency, oscillation amplitude, and steady-state deformation) and can be simulated in seconds. The only calibration step required is a quasi-static cyclic loading test at the pressures of interest. Furthermore, the model offers an effective method for relating functional requirements to design inputs; for example, if a jamming-based system must exhibit nonzero damping at low force inputs, then F_d should be minimized; as Tables 3.1 and 3.2 both show, pressure should be reduced, but not relieved entirely. Thus, the models in this chapter allow designers to predict the dynamic response of jamming structures with nominal effort, as well as meet specific design goals.

Aside from altering impact responses, the capability of laminar jamming structures to tune dynamic responses may have other applications. In fact, the multiple existing methods to actuate jamming structures (e.g., electrostatic [65], elastic [58]) facilitate their integration into diverse systems. For instance, laminar jamming structures may be coupled to dielectric elastomer- and hydrogel-based oscillators [78] to tune their frequency response in real-time, increasing their versatility as acoustic elements and fluidic capacitors. Furthermore, laminar jamming structures may be used as vibration suppression layers in rapidly-actuated soft robots, which are prone to undesired oscillations [79, 80].

3.4 Conclusions

The present chapter has demonstrated that laminar jamming structures can effectively tune dynamic responses in real-world robotic systems while overcoming the limitations of existing variable-impedance mechanisms. Furthermore, through analytical and experimental investigations, the chapter has formulated a lumped-parameter model that can enable designers to rapidly predict the dynamic response of jamming structures and meet specific design requirements. With the models and demonstrations provided here, researchers may take another step

towards building versatile, transformative robots. Future extensions of this part of the thesis will focus on integrating laminar jamming into additional dynamic systems, as well as building an interactive tool in which designers can determine the optimal dimensions and material properties of a laminar jamming structure given ideal dynamic response characteristics.

Chapter 4

Extending Performance Limits with Jamming-Based Composites

This chapter introduces a novel class of tunable composites called jamming-based sandwich structures (also referred to as “sandwich jamming structures”). Sandwich jamming structures are sandwich beams that are divided into numerous layers and activated like laminar jamming structures. As shown here, sandwich jamming structures extend the functional limits of laminar jamming structures, as they have far higher performance-to-mass ratios (e.g., stiffness-to-mass). In addition, their performance exceeds that of other tunable sandwich structures in the literature, and they possess clear advantages of mechanical versatility over standard sandwich structures (e.g., the ability to achieve an arbitrary initial shape).

This work was conducted in collaboration with Buse Aktaş (experimental characterization, optimization, rehabilitation demonstration) and Sarah Ornellas (rehabilitation demonstration). The research also benefited from the assistance of Nikolaos Vasios, who wrote the initial Python script that partially automated the setup of the finite element simulations.

4.1 Introduction

Chapter 2, Chapter 3, and the corresponding published articles [49, 72] have analytically and experimentally demonstrated that laminar jamming structures are compelling tunable-impedance mechanisms. Compared to other mechanisms, they are lightweight, thin, low-cost, and easy-to-fabricate, and they have a high range of stiffness values, resolution of damping values, activation speed, and resistance to bending moments. Nevertheless, there are certain applications in which mass is exceptionally critical. These applications demand laminar jamming structures with even higher performance-to-mass ratios.

For example, in assistive device design, researchers have used laminar jamming structures to conform to the body and immobilize joints [51]. However, laminar jamming structures begin to yield (i.e., exhibit a decreased bending stiffness) after a critical load is exceeded. To prevent yielding, the structures can be thickened, but adding mass to the body increases metabolic energy expenditure [81]. Thus, to improve performance while minimizing energy expenditure, the yield force should be increased while minimizing added mass. As a second example, in quadcopter design, we have used laminar jamming structures to construct landing gear with a tunable impact response. However, the energetic efficiency of aerial vehicles is a monotonically increasing function of the lift-to-drag ratio, and the lift-to-drag ratio decreases with mass [82]. Thus, to maintain performance while maximizing efficiency, the stiffness and damping properties of the landing gear should be preserved while minimizing mass.

To improve the performance-to-mass of laminar jamming structures, we introduce the novel concept of sandwich jamming structures. Standard sandwich structures are composites consisting of thin, stiff facesheets bonded to a thick, low-density core; these structures are frequently used in high stiffness-to-mass systems, such as aircraft [83, 84]. Sandwich jamming structures take the form of standard sandwich structures, but both the facesheets and core are divided into numerous unbonded laminae. As in laminar jamming, when sandwich jamming structures are subject to a pressure gradient (e.g., by enclosing the laminae in an airtight envelope and applying a vacuum (Figure 4.1A)), increased frictional coupling causes a dramatic change in mechanical properties. However, as we show, sandwich jamming structures have

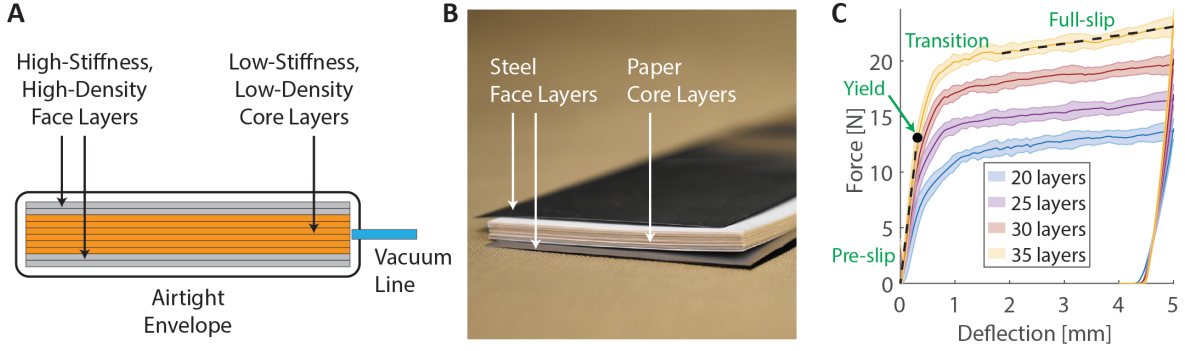


Figure 4.1: Concept and experimental-proof-of-concept of sandwich jamming structures. A) Conceptual diagram of a sandwich jamming structure. Face layers and core layers are enclosed in an airtight envelope connected to a vacuum line. When vacuum is applied, the structure exhibits a dramatic change in mechanical properties. B) Physical prototype of a steel-paper sandwich jamming structure. The layers enclosed in the airtight envelope are depicted. C) Force-versus-deflection curves of steel-paper sandwich jamming structures in 3-point bending at 71kPa vacuum pressure. Curves are shown for different numbers of core layers. Each curve is a mean curve from 2 samples and 10 trials. Shaded error bars delimit ± 1 standard deviation; narrow error bars indicate high repeatability of experimental tests. As with standard laminar jamming structures [49], an initial high-stiffness regime (pre-slip), a yield point, a transition regime, and a final low-stiffness regime (full-slip) are present; these phases are labeled on the curve for 35 layers.

far higher performance-to-mass ratios than laminar jamming structures—specifically, higher stiffness in the jammed state, stiffness range (i.e., the ratio of the jammed to unjammed stiffness), and yield force, all with respect to mass. Such performance also exceeds that of other tunable sandwich structures in the literature [85–89]. Furthermore, sandwich jamming structures possess far greater mechanical versatility than standard sandwich panels. They can transform their stiffness and damping to adapt to the environment (e.g., as fluidic control surfaces), can be molded to an arbitrary initial shape (e.g., when conforming to the body), and can recover their undeformed configuration after yielding (e.g., after impacts).

In this chapter, we first experimentally demonstrate that simple sandwich jamming structures have higher performance-to-mass ratios than laminar jamming analogues. We then present detailed analytical and finite-element models that parametrically describe how and to what extent laminar jamming structures can be improved by converting them to a sandwich jamming architecture. Next, we provide an optimization tool that allows designers to input an arbitrary set of materials and mass-volume constraints and then identify the

highest-performance sandwich jamming structure that can be constructed within those bounds. Finally, we demonstrate the utility of sandwich jamming structures by integrating them into a lightweight, tunable-stiffness wrist orthosis that can reduce muscle activation when turned on and allow nearly-complete range of motion when turned off.

To our knowledge, this work constitutes the first description and implementation of tunable jamming-based sandwich structures, as well as one of the first expositions of jamming-based composite structures in the literature. Furthermore, we provide designers with an analysis and simulation toolkit to relate design parameters of sandwich jamming structures (e.g., the number of layers in the face and core) to performance specifications (e.g, stiffness range), as well as optimize the performance of the structures. Overall, we rigorously demonstrate that sandwich jamming structures can advance the state-of-the-art in lightweight, tunable-impedance structures, and provide useful means to further their development.

4.2 Results

4.2.1 Experimental Proof-of-Concept

To initially validate the concept of sandwich jamming structures, real-world prototypes were constructed and experimentally characterized. Each sandwich structure consisted of faces composed of stiff laminae and cores composed of relatively compliant, lightweight laminae; the structure was enclosed in an airtight envelope and connected to a regulated vacuum source to activate or deactivate jamming.

Satisfying the assumptions of sandwich theory [90], all sandwich jamming structures were constructed such that the small-deformation stress-strain modulus of the face material was at least an order-of-magnitude greater than that of the core material, and the total thickness of the core was at least an order-of-magnitude greater than the total thickness of the faces. Materials were chosen that were low cost, exhibited negligible electrostatic attraction, and could be effectively cut using standard laboratory equipment.

To meet the preceding requirements, sandwich jamming structures were fabricated with

the following material configurations: 1) low-carbon steel face laminae and paper core laminae (Figure 4.1B), 2) low-carbon steel face laminae and low-density polyethylene (LDPE) core laminae, and 3) paper face laminae and polyurethane (PU) foam core laminae. For each material configuration, structures were built using various numbers of core layers. Each sample was placed onto a universal materials testing device, and vacuum was applied. The sample was then loaded in three-point bending, and the force-versus-deflection relationships were measured. Figure 4.1C shows force-deflection curves for the steel-paper configuration, and Figure B.4 shows analogous curves for other configurations. (For additional details on materials, fabrication, and testing, see [Appendix B: Experimental Proof-of-Concept](#)).

Three performance metrics were extracted from the experimental data: jammed stiffness, range (i.e., the ratio of jammed to unjammed stiffnesses), and yield force, all divided by mass. Using previously-reported theory [49], the same performance metrics were calculated for laminar jamming structures consisting of just the face laminae. The performance metrics of the sandwich jamming structures were then divided by the corresponding values for the laminar jamming structures. (For example, the performance metrics for a sandwich jamming structure consisting of 2 layers of steel, 20 layers of paper, and 2 layers of steel were divided by the corresponding values for a laminar jamming structure consisting of 4 layers of steel.) These “improvement ratios” effectively described how much the performance metrics of laminar jamming structures could be improved by adding a core.

Among the three material configurations, the steel-paper configuration exhibited the highest overall performance. For this configuration, the jammed stiffness increased by a factor of up to 2800; furthermore, the maximum stiffness-to-mass, range-to-mass, and yield-to-mass improvement ratios were 176, 65, and 15, respectively. The improvement ratios for all material configurations are tabulated and compared in [Table B.1](#). The results demonstrated that by converting laminar jamming structures to sandwich jamming structures, performance improvements of one to two orders-of-magnitude could be readily achieved, motivating subsequent analysis and optimization.

4.2.2 Analytical Modeling

Analytical modeling was conducted to determine how the performance-to-mass metrics of standard laminar jamming structures could be improved by converting to a sandwich architecture, as well as how these improvements scaled with critical design parameters. For practical relevance, it was first considered what constraints should be analytically imposed when converting laminar jamming structures to a sandwich configuration.

For designers, three major constraints should be considered: 1) an “equal-material” constraint, in which *all* the sheets of the laminar jamming structure are used as face laminae for the sandwich jamming structure, and core laminae (which are compliant and typically far less expensive) can be added without constraint (Figure 4.2A), 2) an “equal-mass” constraint, in which a *subset* of the sheets of the laminar jamming structure are used as face laminae for the sandwich jamming structure, and core laminae can be added with the constraint that the total *mass* of the sandwich must equal the mass of the laminar jamming structure (Figure B.2A), and 3) an “equal-volume” constraint, in which a *subset* of the sheets of the laminar jamming structure are used as face laminae for the sandwich jamming structure, and core laminae can be added with the constraint that the total *volume* of the sandwich must equal the volume of the laminar jamming structure (Figure B.3A). For sake of brevity, the following analysis considers an equal-material constraint; in [Appendix B: Analytical Modeling](#), equal-mass and equal-volume constraints are investigated as well.

In order to determine how the performance-to-mass metrics of standard laminar jamming structures could be improved by converting to a sandwich architecture (with an equal-material constraint), the performance-to-mass of each type of structure was first calculated. As in the preceding experimental investigation, three performance-to-mass metrics were computed: bending stiffness, range, and yield force, all divided by mass. These metrics were calculated for standard laminar jamming structures using a previously-reported application of Euler-Bernoulli beam theory [49]. The metrics were then calculated for sandwich jamming structures using sandwich beam theory [83], an extension of Timoshenko beam theory that also assumes that 1) the beam consists of two faces and a core, 2) the faces are much thinner than the core,

and 3) the core is much more compliant than the faces. Ratios were then computed of the performance-to-mass metrics of the sandwich jamming structure to the corresponding metrics of a standard laminar jamming structure. The ratios were expressed as functions of critical non-dimensional design parameters. Step-by-step derivations are provided in [Appendix B: Analytical Modeling](#); a highly abbreviated version follows here.

From sandwich theory, the jammed stiffness of a sandwich jamming structure is approximately $E_f b \frac{4c^2 f + 4c f^2 + f^3}{16}$, where E_f is the elastic modulus of the face material; b is the width; and c and f are the total thickness of the core and face, respectively. From beam theory, the jammed stiffness of a laminar jamming structure is $\frac{EbH^3}{12}$, where E is the elastic modulus of the layers and H is the total thickness. These expressions are critical for deriving the stiffness-to-mass improvement ratio, which is

$$\left(\frac{k_b}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (4.1)$$

where $\left(\frac{k_b}{m}\right)^*$ is the dimensionless ratio of the bending-stiffness-to-mass of the sandwich jamming structure to that of the laminar jamming structure, $\frac{c}{f}$ is the ratio of the total thickness of the core to that of the faces; and $\frac{\rho_c}{\rho_f}$ is the ratio of the density of the core material to the density of the face material.

To calculate the range of a jamming structure, the unjammed stiffness must be computed as well. From beam theory, the unjammed stiffness of a sandwich jamming structure is $b \frac{E_c n_c h_c^3 + E_f n_f h_f^3}{12}$, where E_c is the elastic modulus of the core; n_c and n_f are the total number of layers in the core and faces, respectively; and h_c and h_f are the thickness of each core and face layer, respectively. The unjammed stiffness of a laminar jamming structure is $\frac{Ebnh^3}{12}$. These expressions are critical for deriving the range-to-mass improvement ratio, which is

$$\left(\frac{r}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\frac{E_c}{E_f}\left(\frac{h_c}{h_f}\right)^2\frac{c}{f} + 1\right)\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (4.2)$$

where $\left(\frac{r}{m}\right)^*$ is the ratio of the range-to-mass of the sandwich jamming structure to that of the laminar jamming structure, and $\frac{E_c}{E_f}$ is the ratio of the elastic modulus of the core material to that of the face material.

Finally, to calculate the yield of a jamming structure, the maximum induced shear stress must be equated with the maximum allowable shear stress (as limited by friction and pressure). For a sandwich jamming structure, the yield force is approximately $2bc\mu_c P$, where μ_c is the coefficient of friction of the core material and P is the vacuum pressure; for a laminar jamming structure, the yield force is $\frac{4}{3}bH\mu P$, where μ is the coefficient of friction of the layers. These expressions are critical for deriving the yield-to-mass improvement ratio, which is

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1}{1} \quad (4.3)$$

where $\left(\frac{F_{crit}}{m}\right)^*$ is the ratio of the critical yield force of the sandwich jamming structure to that of the laminar jamming structure, and $\frac{\mu_c}{\mu_f}$ is the ratio of the coefficient of friction of the core material to that of the face material.

Figure 4.2 provides contour maps that illustrate the functional dependence of the improvement ratios on the non-dimensional design parameters. As shown in the plots, the improvement ratios are nonlinear functions of the design parameters, with notable covariance. By converting a laminar jamming structure to a sandwich jamming structure, stiffness-to-mass and range-to-mass can both be readily improved by over four orders of magnitude, and yield-to-mass can improved by over two orders of magnitude. These improvements can be achieved by maximizing the the total thickness ratio and friction ratio, and minimizing the density ratio, elastic modulus ratio, and layer thickness ratio.

4.2.3 Finite Element Simulations

To corroborate analytical predictions, finite element simulations of sandwich jamming structures were conducted. Each layer was modeled as a 2-dimensional body, and frictional contact was prescribed at the interfaces between adjacent layers. Vacuum pressure was applied, and the structures were loaded in 3-point bending (Figure 4.3A-C). A static implicit solver was used. The simulations were executed over a range of design parameters that could be achieved with common laboratory materials (e.g., standard metals and plastics). Since the simulations were static rather than dynamic, inertia was negligible; thus, density ratio was not varied,

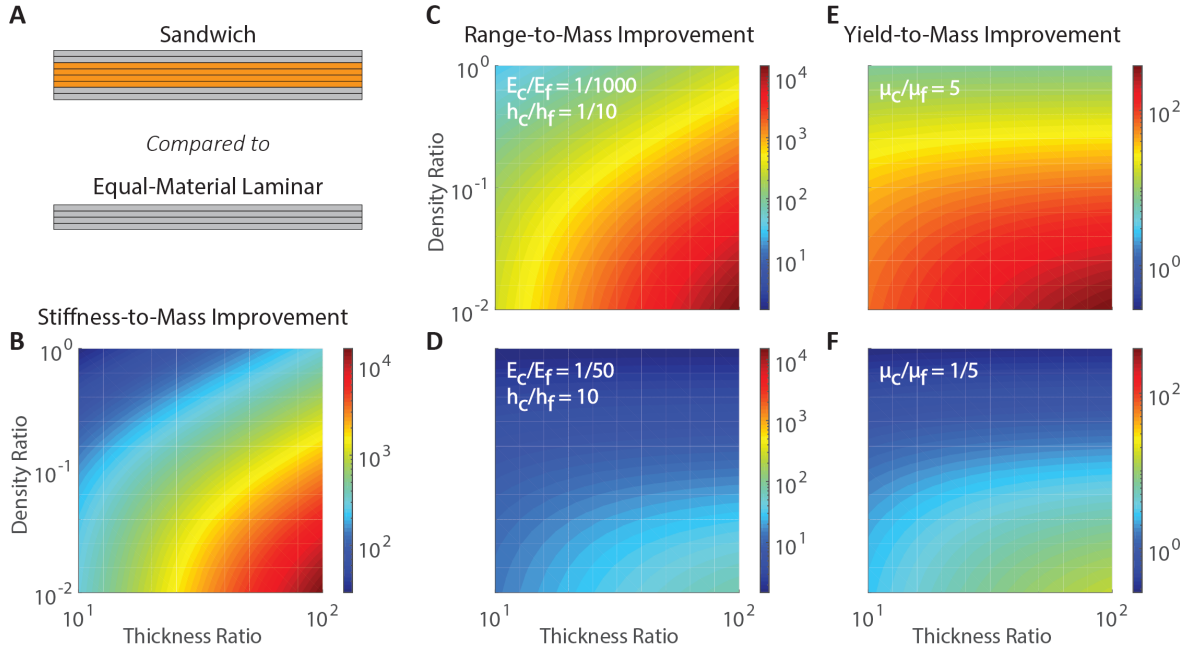


Figure 4.2: A) Diagram of an equal-material laminar jamming structure, which is used for comparison. B) Contour maps of analytical predictions. Each plot illustrates performance improvement as a function of density ratio ($\frac{\rho_c}{\rho_f}$) and total thickness ratio ($\frac{c}{f}$). Range-to-mass improvement is also a function of $\frac{E_c}{E_f}(\frac{h_c}{h_f})^2$, and yield-to-mass improvement is also a function of $\frac{\mu_c}{\mu_f}$. Thus, two plots are provided for each of these performance metrics to show variation with these additional parameters. Design parameters are varied over a range representative of real-world limits. Note that both stiffness-to-mass and range-to-mass can be improved by four orders of magnitude, and yield-to-mass can be improved by two orders of magnitude.

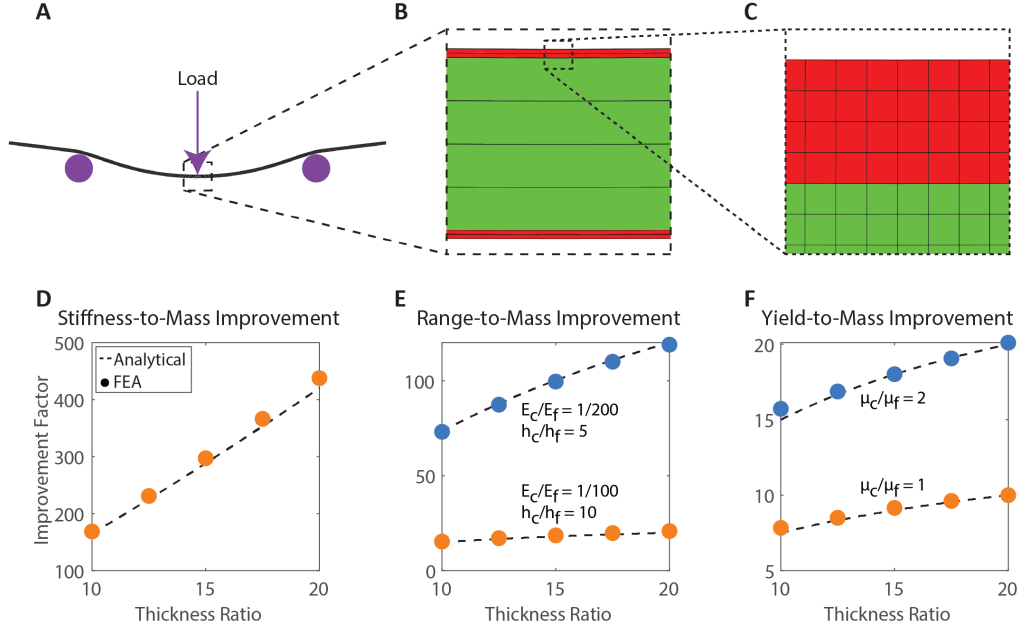


Figure 4.3: Comparison of analytical predictions to finite element results. Dashed lines denote analytical predictions, and filled circles denote finite element results. The analytically-predicted stiffness-to-mass, range-to-mass, and yield-to-mass improvement ratios were all closely corroborated by finite element values. The minimum coefficient of determination (R^2) between any analytical and finite element data set was 0.992 for the yield-to-mass improvement ratios with $\frac{\mu_c}{\mu_f} = 2$.

as it did not affect the output. From the simulations, force-versus-deflection curves were extracted. As with the experimental data, the stiffness, range, and yield forces were extracted from these curves. Simulation procedures, specific parameters, and raw simulation output are all provided in [Appendix B: Finite Element Simulations](#).

Figure 4.3D-F compares analytical predictions and finite element results. As illustrated, finite element simulations corroborated analytical predictions with exceptional fidelity. In fact, the minimum coefficient of determination (R^2) between analytical predictions and finite element results was greater than 0.99, indicating that the analytical model exhibited excellent accuracy relative to a sophisticated computational reference.

4.2.4 Optimization

The preceding analysis and simulations determined exactly how and to what extent given laminar jamming structures can be improved by converting them to a sandwich architecture

(i.e., by adding a core). However, for a designer who is building sandwich jamming structures from the ground-up, it is also important to identify the sandwich jamming structure with the best-possible performance-to-mass ratios, rather than improvement ratio over a given laminar jamming structure. Furthermore, the designer may be constrained by mass, volume, and a particular set of real-world materials.

This problem is analytically challenging for two major reasons. First, although performance improvement ratios are dimensionless and can be expressed as a function of a small number of non-dimensional parameters (e.g., $\frac{\rho_c}{\rho_f}$), performance metrics themselves are dimensional and cannot be analogously simplified (e.g., the absolute magnitudes of both ρ_c and ρ_f may be critical). Thus, maximizing performance requires investigation of a much larger parameter space, which becomes difficult to plot and intuitively understand. Second, real-world materials do not allow arbitrary variations of material parameters (e.g., ρ , E , μ) with respect to one another; as a result, many discrete constraints must be imposed.

The preceding problem was effectively addressed through numerical optimization. A software routine first allows designers to input an arbitrary set of available materials, as well as any mass, volume, and layer-thickness constraints relevant to their application. For each pair of materials, the routine optimizes the geometry of the face, core, and individual laminae using a constrained nonlinear optimization algorithm in order to maximize the stiffness-to-mass, range-to-mass, and yield-to-mass ratios of the structure. Finally, for each of these performance metrics, the routine determines the best-performing structure. Full details of the optimization process are provided in [Appendix B: Optimization](#).

A case study illustrates the results. Four materials were input into the software routine (low-carbon steel, paper, LDPE, and PU foam), along with their material properties and minimum available thicknesses (Table [B.3](#)). Mass constraints (total mass ≤ 47 g) and volume constraints (width = 50 mm, length = 100 mm, and total height ≤ 7.5 mm) were applied. The routine identified five material pairs (i.e., steel-paper, steel-LDPE, steel-foam, paper-foam, and LDPE-foam) that satisfied the assumptions of sandwich theory and recommended construction guidelines (i.e., $E_c \gg E_f$) [[83,90](#)]. For the range-to-mass ratio, Table [4.1](#) lists

Table 4.1: Results from optimization case study for range-to-mass ratio. The first- and second-best performing material configurations were steel-foam and steel-LDPE, respectively.

Material Configuration	Optimized Parameter				
	n_c	$h_c[m]$	n_f	$h_f[m]$	Range-to-mass $[\frac{1}{kg}]$
Steel-Paper	70	1.0e-4	8	5.0e-5	5.1e5
Steel-LDPE	73	1.0e-4	2	5.0e-5	1.4e6
Steel-Foam	9	7.9e-4	2	5.0e-5	2.2e6
Paper-Foam	8	7.9e-4	6	1.0e-4	5.2e5
LDPE-Foam	8	7.9e-4	6	1.0e-4	8.0e4

the best-performing sandwich structures for each material configuration, as well as their corresponding geometric properties. Table B.4 and Table B.5 provide analogous data for the stiffness-to-mass and yield-to-mass ratios, respectively.

In practical scenarios, mass and volume constraints may quickly change. To provide designers with intuition about how a change of constraints can influence optimal geometries, the software routine can also generate contour maps that show how performance metrics vary with geometry, with constraints directly illustrated on the maps. Figure 4.4 shows an example of these contour maps for the steel-paper configuration. To use these maps, the designer first chooses mass and/or volume constraints on the map. The designer then looks for the highest value of the performance metric (from the color bar) that lies to the left of the constraint lines. The optimal geometry is given by the x - and y -coordinates of this value, which denote the number of core and face layers, respectively. This process can be easily repeated for another set of mass and volume constraints as desired.

4.2.5 Demonstrations

Sandwich jamming structures may be useful in a range of applications, including assistive devices, vehicles, and deployable structures. We explored the first category by constructing a wrist orthosis with a sandwich jamming structure as a tunable-stiffness element. For patients with wrist injuries, a tunable-stiffness orthosis may reduce muscle activation during static

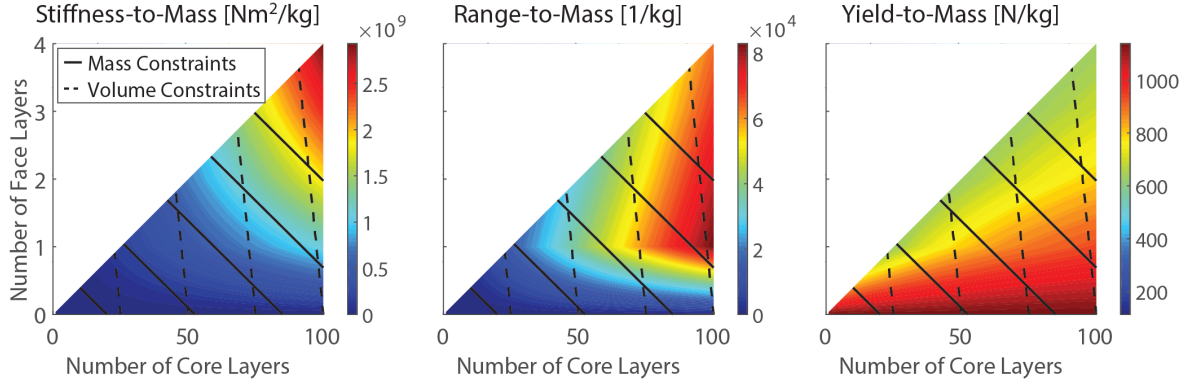


Figure 4.4: Contour maps for optimization of steel-paper sandwich structures. The plots illustrate the variation of optimal performance values and corresponding geometries (i.e., number of core and face layers) with applied constraints (i.e., maximum mass and/or volume). The mass and volume constraints are shown in solid and dashed lines, respectively, on each graph; from left-to-right, the mass constraints are {8, 21, 34, 47, 60}g, and the volume constraints are a maximum height of {2.5, 5, 7.5, 10}mm.

weight-bearing tasks (e.g., carrying grocery bags), but enable flexibility during non-strenuous dynamic tasks (e.g., driving). A sandwich jamming structure presents a highly compelling tunable-stiffness structure for this application, as it not only has high stiffness-to-mass, range-to-mass, and yield-to-mass ratios, but is also thin, conformable, and rapidly activated. Thus, we built a sandwich-jamming wrist orthosis that was intended to facilitate isometric hold tasks when the structure was jammed, but allow full range-of-motion when unjammed.

The orthosis consisted of an elastic sleeve for the hand that was sewn to a non-slip fabric sleeve for the arm (Figure 4.5A); the composite sleeve was secured securely and comfortably to the arm via hook-and-loop (i.e., Velcro) straps. A sandwich jamming structure was attached to the palmar side of the hand via another strap and was allowed to slide freely along the long axis of the arm within a pocket on the brace. The optimization software routine was used to determine the materials and geometry of a high-performance sandwich jamming structure that satisfied specific mass and volume constraints; the sandwich jamming structure consisted of 6 layers of low-carbon steel, 30 layers of paper, and 6 layers of steel. Additional details on the optimization and fabrication process are provided in [Appendix B: Demonstrations: Optimization and Fabrication](#).

To evaluate the performance of the brace during weight-bearing activities, an isometric

hold task was conducted on human subjects. The orthosis was fastened onto each subject, and electromyography (EMG) electrodes were located adjacent to the flexor carpi ulnaris muscle to measure muscle activation. Each subject was requested to bend their elbow to 90 degrees and freely rest it on a flat surface. A weight was suspended from their hand, and the subject was requested to keep their wrist flat with minimal effort. Muscle activation was recorded. The test was conducted with 1) no orthosis, 2) the sandwich jamming brace in the unjammed state, and 3) the brace in the jammed state.

Figure 4.5D-E compares muscle activation profiles across the three conditions for a representative subject. As illustrated, average muscle activation in the unjammed state was less than that in the no-brace state, indicating that wearing the brace did not impose significant demands on the musculoskeletal system. Moreover, average muscle activation was approximately 70% less in the jammed state than the no-brace state, demonstrating that the brace could effectively reduce musculoskeletal demand when desired. Similar muscle activation profiles for another subject are provided in Figure B.6.

To evaluate the flexibility of the brace during non-weight-bearing activities, a range-of-motion test was also conducted (Figure 4.5B-C). During these tests, subjects were asked to flex and extend their wrists to the maximum angle that they still perceived as comfortable. The test was executed for the no-brace and unjammed conditions. It was found that subjects in the unjammed state achieved a minimum of 84% of the range of motion that they could achieve in the no-brace state during extension, and a minimum of 60% during flexion. Thus, significant range of motion was preserved. Additional experimental details for the isometric hold and range-of-motion tests are provided in [Appendix B: Demonstrations](#).

4.3 Discussion

In this chapter, we have introduced the concept of tunable jamming-based sandwich structures through experiments, models, and demonstrations. We first experimentally showed that simple sandwich jamming structures can achieve notably higher performance than laminar jamming structures. We then presented an analytical model that describes how the performance-to-mass

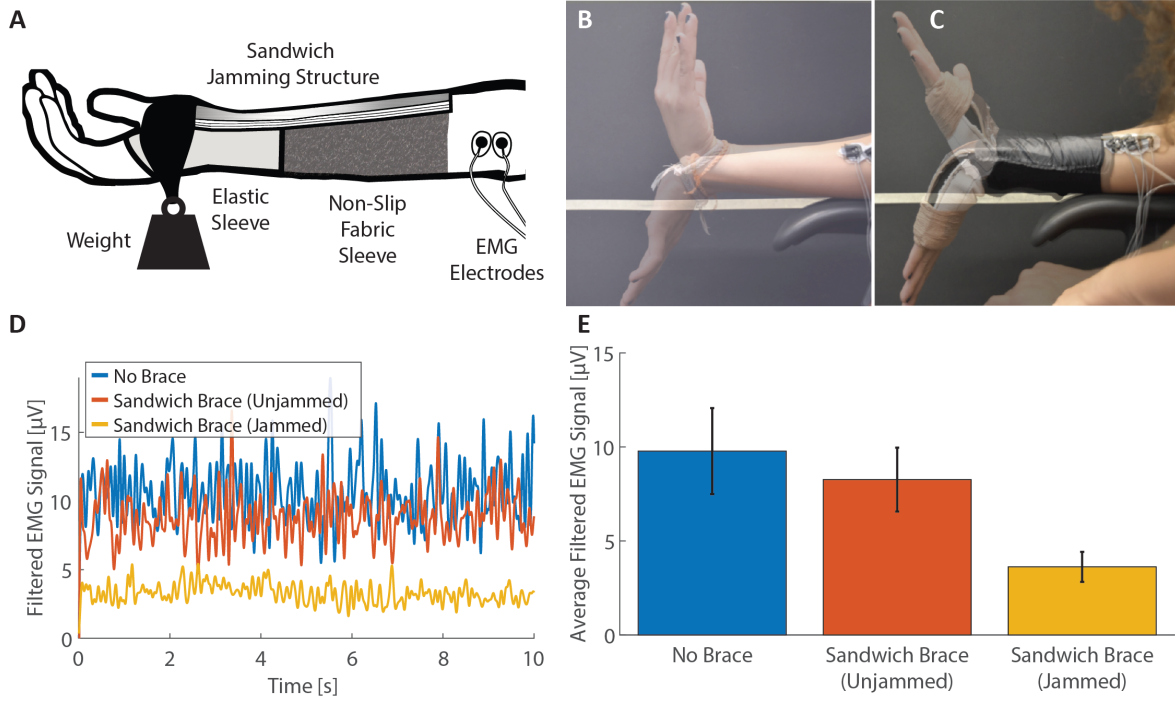


Figure 4.5: Overview of sandwich-jamming wrist orthosis. *A)* Diagram of orthosis. For clarity, the hook-and-loop straps and the pocket constraining the sandwich jamming structure are not depicted. *B-C)* Range-of-motion trials of a human subject in the no-brace and unjammed conditions, respectively. *D-E)* Comparison of EMG profiles of a human subject during the no-brace, unjammed, and jammed conditions. The average EMG signal for the unjammed condition was slightly less than that of the no-brace condition, and the average signal for the jammed condition was approximately 70% less than that of the no-brace condition. Error bars on the bar plot denote ± 1 standard deviation.

of laminar jamming structures can be improved by converting to a sandwich jamming architecture. This analytical model explicitly relates critical design parameters (e.g., core-to-face thickness ratios) to performance metrics and was corroborated by finite-element simulations. Next, we provided an optimization tool that can identify the best-possible sandwich jamming configuration given an arbitrary set of materials and mass-volume constraints. Finally, we demonstrated the utility of sandwich jamming structures by showing that a sandwich-jamming wrist orthosis can reduce muscle activation in isometric hold tasks when jammed, while allowing significant range of motion when unjammed. Collectively, our models, experiments, and demonstrations provide a full analytical and empirical description of a novel structure that advances the state-of-the-art in tunable-impedance mechanisms.

Our work offers several major contributions. As a first contribution, to our knowledge, this paper provides the first exposition of the concept of tunable jamming-based sandwich structures. This concept also serves as one of the first explorations of tunable jamming-based composites in the literature. Previous studies have combined discrete granular and laminar jamming elements [54] and integrated sensing and actuation components into jamming structures [55, 91], but did not investigate how to achieve high performance-to-mass, which is a trademark capability of composite materials and structures.

Second, we showed that our specific implementations of sandwich jamming structures were able to achieve exceptional mechanical properties. The structures experimentally outperformed the stiffness-, range-, and yield-to-mass of laminar jamming analogues by one to two orders of magnitude. Furthermore, these sandwich jamming structures exceeded the performance of other tunable sandwich structures in the literature. Existing tunable sandwich structures have typically been constructed with one of the following features: 1) a core containing electrorheological fluid [85], 2) a core containing magnetorheological fluid [87], 3) a core containing shape-memory material [86, 89], and 4) facesheets that are electrostatically bonded to the core [88]. Among these structures, experimentally validated stiffness ranges are typically well below 2, and the highest range found in the literature was 18 [88]. Our study experimentally demonstrated that a simple steel-paper sandwich jamming structure could

achieve a stiffness range of 42 in a lightweight form. In contrast to previous embodiments, the structure was low cost, could be rapidly activated, and did not require high voltages that could compromise safe interaction with humans.

As a third contribution, this paper provides a detailed analytical framework for designers to construct and optimize sandwich jamming structures to meet design requirements. Given an existing laminar jamming structure, the improvement ratios and associated contour maps inform designers exactly how the performance metrics of the structure (i.e., stiffness-to-mass, range-to-mass, and yield-to-mass) can be improved by adding a core and adjusting critical design parameters (i.e., thickness ratio, density ratio, elastic modulus ratio, layer number ratio, and friction ratio); these predictions show that improvement ratios of several orders of magnitude may be readily achievable. Moreover, the optimization code and associated contour maps allow designers to select an arbitrary set of face and core materials and rapidly determine the highest-performing structure that can meet their mass and volume constraints. Thus, the analysis allows designers to deterministically improve existing structures, as well as design optimal structures from the ground-up.

The demonstrations comprise the final contribution of the paper, as they provide feasible solutions for a challenging engineering problem. In rehabilitation, previous efforts to determine whether wrist orthoses can reduce muscle activation have been inconclusive, with various studies showing moderately positive [92], negative [93], and neutral results [94]. Our study demonstrated that a sandwich-jamming orthosis markedly reduced muscle activation on a small population of participants when jammed, providing a compelling basis for further investigation. Moreover, in contrast to wrist orthoses that have previously been reported, the sandwich-jamming orthosis is highly tunable, allowing significant range of motion and causing no notable increase in muscle activation when unjammed.

Although sandwich jamming structures may have intrinsically high damping, shear stiffness, damping range, and shear stiffness range (all with respect to mass), the improvement ratios and contour maps for these performance metrics are not provided in this paper for sake of brevity. Future work will not only focus on investigating these performance metrics, but also pushing

the performance boundaries of tunable-impedance mechanisms even further. Two immediate improvements in the performance-to-mass of sandwich jamming structures can be achieved by using composite materials as facesheets (e.g., carbon-fiber-reinforced polymers or fiberglass), as well as increasing the porosity of the core layers (e.g., by laser-cutting honeycomb patterns into the layers). Furthermore, to maximize portability of the sandwich jamming structures while preserving human safety, jamming can be activated with non-fluidic and non-electrostatic methods, such as tightening an elastic mesh around the layers [58]. These improvements can augment the remarkable mechanical properties and utility of sandwich jamming structures even further, and ultimately lay a foundation for their commercial adoption in wearable robotic devices, modern vehicles, and rapidly deployable construction.

4.4 Conclusion

This chapter proposed the novel concept of tunable, sandwich-based jamming structures. These structures were experimentally shown to have even higher performance-to-mass than laminar jamming structures and existing tunable sandwich structures, as well as clear versatility advantages over traditional sandwich structures. Analytical models and finite element simulations were provided that allow designers to deterministically improve the performance-to-mass of laminar jamming structures by converting them to a sandwich architecture. A software routine was presented that also allowed designers to optimize sandwich jamming structures given arbitrary material, mass, and volume constraints. Finally, the utility of sandwich jamming structures was demonstrated through the implementation of a sandwich-jamming wrist orthosis, which significantly reduced muscle activation of human subjects in the “on” state while having negligible impact on activation in the “off” state. Overall, sandwich jamming structures were shown to extend the performance boundaries of existing tunable-impedance mechanisms, setting the stage for the next generation of ultra-light tunable-impedance devices. Future work will focus on improving performance further through the integration of composite materials and portable jamming-activation mechanisms.

Chapter 5

Conclusions

In this chapter, the contributions of this thesis will be summarized. Next, theoretical and applied areas for investigation will be outlined, and a comprehensive list of potential applications will be presented. Finally, a framework will be proposed for generalizing this field of research (i.e., jamming-based mechanisms for tunable mechanical behavior).

5.1 Contributions

As described in the introduction, the goal of this thesis was to bridge the gap between the physical paradigms of soft robotics and traditional rigid robotics. To accomplish this goal, we aimed to identify and rigorously explore a variable-impedance mechanism that could enable soft robots to behave like traditional rigid systems on command, as well as allow the soft robots to rapidly return to their original state. The laminar jamming phenomenon was identified as a particularly promising mechanism and was investigated in great detail.

Overall, this thesis has demonstrated how the laminar jamming phenomenon works; how laminar jamming structures can transform the stiffness, damping, kinematics, and dynamic response of robotic systems; how designers can relate design parameters to performance metrics; and how the performance of laminar jamming structures can be pushed past the state-of-the-art. To the best of our knowledge, this work is the first to provide the following:

1. A detailed understanding of the laminar jamming phenomenon (through analytical models that describe deformation, energy dissipation, stiffness, damping, and slip propagation during major all phases of loading)
2. Accurate and rapid prediction of the static and dynamic behavior of arbitrary laminar jamming structures (through finite-element and lumped-parameter models)
3. Application of laminar jamming to achieve shape-locking behavior and tunable impact responses (through physical prototypes)
4. A mechanism of any kind to achieve a variable-kinematics function in soft robots (through finite-element models and physical prototypes), contemporaneous with one other research group [35, 36])
5. Design tools (i.e., scaling relations, design plots, and optimization routines) to allow designers to customize laminar jamming structures to meet functional requirements
6. The novel concept of tunable, jamming-based sandwich structures, as well as the demonstration of their utility (through analytical models and physical prototypes)

More specifically, these contributions were divided across the chapters as follows:

- **Chapter 2:** An analytical model of laminar jamming that provides understanding of the laminar jamming phenomenon and predicts static behavior of 2-layer jamming structures during pre-slip, full-slip, and the transition regime; finite-element simulations that accurately predict static behavior of many-layer jamming structures; scaling relations that relate critical design parameters of laminar jamming structures to performance metrics; and the use of laminar jamming structures to achieve shape-locking and variable-kinematics functions.
- **Chapter 3:** Finite-element simulations that accurately predict static hysteresis behavior of many-layer jamming structures (which, as discussed in the chapter, capture dynamic behavior as well); a lumped-parameter model that rapidly predicts dynamic behavior of

many-layer jamming structures and provides designers with intuition about jamming dynamics; and the use of laminar jamming structures to achieve tunable impact responses.

- **Chapter 4:** Experimental evidence that sandwich jamming structures outperform laminar jamming structures and existing tunable sandwich structures; analytical and finite-element models that predict theoretical performance improvement of structures relative to standard laminar jamming structures; design plots that relate design parameters to performance improvement; optimization routine that identifies best-performing sandwich structures subject to design constraints; and a physical prototype of a wearable device that uses sandwich jamming structures to selectively reduce muscle activation.

5.2 Future Work

This thesis has presented work in both analytical mechanics and robotic structural design. Similarly, avenues for future work can be divided between these two categories. The following subsections list particularly compelling opportunities for research in these areas:

5.2.1 Analytical Investigations

- **Analytical modeling of many-layer jamming structures:** In Chapter 2, detailed analytical models were provided for 2-layer laminar jamming structures, and in the appendix to Chapter 2, a procedure was outlined to derive analytical models for n -layer structures, where n is an arbitrary integer greater than 2. However, executing the procedure may be algebraically taxing. To approximate the mechanical behavior of many-layer structures, it may be worth deriving a continuum model (e.g., a model based on those of elastoplastic structures with growing regions of plastic deformation).
- **Finite-element simulations of many-layer jamming structures:** As described in Chapter 2, the execution time for the finite-element simulations of multi-layer laminar jamming structures scaled approximately linearly with the number of layers due to the increasing number of contact surfaces. As mentioned in the appendix to Chapter 2, a

many-layer jamming structure could be approximated as a single crystal with a single slip system, and existing simulation packages for crystal plasticity could be used to conduct finite element analysis with much shorter execution time.

- **Finite-element and experimental validation of slip propagation:** The analytical model in Chapter 2 predicted how slipped regions would grow and propagate throughout a 2-layer laminar jamming structure in cantilever bending. Finite-element analysis could be used to corroborate the analytically-predicted growth of slipped regions, and high-resolution imaging techniques could be used to experimentally validate these predictions.
- **Investigation of curvature reversal phenomenon:** As described in the appendix to Chapter 2, laminar jamming structures exhibit a fascinating “curvature reversal” phenomenon when subject to high loads. Whereas the curvature of a standard beam in 3-point bending remains greater than or equal to zero throughout the length of the beam, the curvature of a laminar jamming structure in 3-point bending may actually change sign along its length. Note that curvature reversal essentially allows each layer of the jamming structure to preserve its initial length during bending, but it requires slip between the layers in order to occur. Thus, it may be thought of as the consequence of a competition between strain energy and dissipated energy.

The strain energy of a jamming structure during pre-slip scales with $EI\kappa^2L$, where E is the elastic modulus, I is the area moment of inertia, κ is the curvature, and L is the length. The dissipated energy of a jammed structure during full-slip scales with $\mu PbHL$, where μ is the coefficient of friction, P is the vacuum pressure (i.e., the absolute pressure below ambient pressure), b is the width, and H is the total height. The ratio of strain energy to dissipated energy then scales with $\frac{EH^2\kappa^2}{\mu P}$; curvature reversal may become significant as this ratio becomes large. Analytical, finite-element, and experimental investigations can all be conducted to validate this back-of-the-envelope prediction and quantify the growth and magnitude of curvature reversal in jamming structures.

Note that curvature reversal is not just an analytically-interesting phenomenon. It may also have great practical relevance, as jamming structures that exhibit significant curvature reversal may be difficult to conform to arbitrary developable surfaces (e.g., in wearable applications).

- **Analytical modeling of shear stiffness and damping of sandwich jamming structures:** In Chapter 4, performance improvement ratios were calculated for bending-stiffness-to-mass, range-to-mass, and yield-to-mass. As mentioned in the Discussion section, performance improvement ratios for shear-stiffness-to-mass and damping-to-mass can readily be calculated as well, but were not included for sake of brevity. For certain applications, it may be important to derive these ratios. For instance, for sandwich jamming structures with highly flexible cores, shear stiffness is critical for determining deflection under load, and for sandwich jamming structures used for shock absorption, damping is critical for determining how much energy is dissipated.
- **Analytical modeling of sandwich jamming structures with porous cores:** Porous structures (e.g., honeycomb structures and cellular solids) are frequently used in the core of modern sandwich panels, as these constituents have high transverse normal stiffness and low weight. However, in Chapter 4, only fully solid materials were considered for the core. Using existing existing models for honeycombs and cellular solids, analytical models could be developed to determine how the performance of sandwich jamming structures can be further improved by using porous structures in the core.
- **Analytical modeling of hybrid jamming structures:** Although granular and laminar jamming has attracted much interest in the robotics and mechanics literature, hybrid jamming (i.e., the combination of multiple types of jamming within a single structure) has been largely unexplored. In our preliminary investigations of these structures, clear practical benefits have yet to be determined, but analytically interesting behavior has been identified. For instance, sandwich jamming beams have been constructed that consist of laminar faces and fiber cores, with the fibers oriented along the width of the

structure. When the structure is sheared in the jammed state, it discretely snaps to a kinematically-favorable deformed state; this deformed state is visually reminiscent of metallic crystals that have undergone twinning deformations.

5.2.2 Design Investigation

Relative to other variable-impedance mechanisms, laminar jamming structures have numerous advantages. They are thin, conformable, lightweight, low-cost, and highly versatile (i.e., can be used to alter stiffness, damping, kinematics, and dynamic response). Furthermore, they can support high bending loads, can achieve an arbitrary initial shape (by jamming it in a desired configuration), and can be reset after yielding (by unjamming it). In addition, vacuum-activated implementations of laminar jamming structures are easy to fabricate, rapidly activated and deactivated, and human-safe.

On the other hand, state-of-the-art designs for laminar jamming structures also have several characteristics that can be improved. Aspects deserving further design investigation are as follows:

- **Portability:** The vacuum-activated implementations of laminar jamming structures described in the previous chapters were all connected to physically stationary laboratory vacuum sources that were regulated by manual or digital regulators. Currently-available low-cost portable vacuum pumps do not achieve high vacuum and are noisy when activated, limiting their functionality and convenience in mobile, human-interactive applications (e.g., wearable devices).

Portability of vacuum-activated laminar jamming structures could be improved by designing higher-power miniaturized vacuum pumps with increased noise suppression, or by using a vacuum reservoir (i.e., an evacuated pressure vessel) that could be opened or closed using a solenoid valve. Portability of laminar jamming structures could also be improved by adopting alternative actuation mechanisms. Two promising human-safe mechanisms are 1) a motor that pulls a tendon attached to a braided mesh that contains flexible laminae; the mesh tightens to jam the layers [58], and 2) spring clips compressing

an airtight envelope that contains flexible laminae; the layers are passively jammed, and compressed air is used to actively unjam them [49]. Another possible mechanism could consist of a motor that pulls tendons attached to two mirrored four-bar linkages; the linkages flatten to jam the layers.

- **Stiffness resolution:** As described in Chapter 2, as long as nonzero vacuum pressure is applied, the specific magnitude of vacuum pressure applied to a laminar jamming structure does not affect its stiffness in the jammed state; this stiffness is simply equal to the stiffness of a cohesive structure composed of the constituent materials. Thus, the stiffness of a laminar jamming structure is binary. In order to achieve improved stiffness resolution, multiple jamming structures can be stacked on top of each other and activated independently [49, 50].

Nevertheless, to achieve fine resolution, the preceding method may be impractical due to the number of independent inputs or valves to control. Another possible method that does not require additional inputs could be to micromachine or laser-etch the surfaces of the laminae such that the contact area between the layers continuously increases as vacuum pressure is increased. Thus, the percentage of the structure in the jammed state increases as well, augmenting the overall stiffness.

- **Maximum yield force:** The yield force of a vacuum-activated laminar jamming structure scales linearly with the coefficient of friction and vacuum pressure. For commonly-available materials, the coefficient of friction is limited to a maximum value of between 1 and 1.5. Furthermore, vacuum pressure is the pressure difference between the ambient pressure outside the airtight envelope and the pressure inside the envelope; thus, it is limited to the absolute magnitude of the ambient pressure itself, which is approximately $101kPa$ at sea level.

For certain applications (e.g., wearable devices that must resist high forces without deforming), a higher maximum yield force may be desired. A few distinct solutions may be considered. First, the application may be limited to environments in which ambient

pressure is high (e.g., underwater). Second, the surfaces may be micromachined (e.g., with a shallow sawtooth profile [50]) or laser-etched to increase the maximum yield force when jammed, while having a negligible effect on the resistance to sliding when unjammed. Third, as described in *Portability*, alternative activation mechanisms can be used. Whereas the maximum yield-force of a vacuum-activated mechanism is limited by physical laws, the yield-force limits imposed by these alternative mechanisms can typically be adjusted (e.g., by increasing the size of a motor or the stiffness of a spring clip).

5.3 Potential Applications

In this thesis, the potential of laminar jamming structures to transform mechanical behavior has been harnessed in a wide diversity of applications. Specifically, these structures have allowed shape-locking in soft actuators, enabled variable kinematics in robotic fingers, tuned collision responses in soft structures, modulated the landing dynamics of quadcopters, and reduced muscle activation in human subjects wearing wrist orthoses. Nevertheless, these applications may just be scratching the surface of possibilities. The following is a list of unexplored applications that may be particularly compelling for investigation in research and product development:

- **Robotic surgery:** Laminar jamming and granular jamming structures have previously been used for stiffness tuning of surgical and continuum manipulators [27, 42, 53, 54]. Stiffness tuning can allow the manipulator to conform to delicate obstacles (e.g., internal organs) during navigation, while also allowing the end-effector to apply high directed forces (e.g., during the cutting of tissue) when the target has been reached. The configuration of the manipulator may occasionally need to be precisely controlled; however, control can be challenging due to the number of passive degrees of freedom. Using laminar jamming to achieve variable kinematics in the manipulator can greatly reduce the number of passive degrees of freedom when desired and enable precise active

control at a small number of joints.

- **Musculoskeletal rehabilitation:** Chapter 4 of this thesis has demonstrated that using sandwich jamming structures for stiffness tuning of wrist orthoses can reduce muscle activation during isometric hold tasks when the orthosis is jammed, while preserving range of motion when unjammed. However, the damping function of jamming structures may also be useful, particularly for lower-limb orthoses. For example, during walking on flat ground, the muscles surrounding the knee joint act to dissipate energy during specific segments of the gait cycle. A sandwich jamming structure can be used in the post-yield regime to dissipate energy during these segments, but can be deactivated to provide negligible resistance during others.
- **Protective equipment:** Bulletproof vests typically consist of a network of high-stiffness, high-strength fibers (e.g., aramid) that slide upon experiencing a high-energy impact, dissipating energy through friction. For non-critical applications, low-cost personal protective gear could be made of laminar jamming structures in which the layers slide upon experiencing low-energy impacts.
- **Robotic locomotion:** Crawling locomotion with compliant, continuously-deforming legs is highly effective for navigating obstacles and traversing uneven terrain. On the other hand, upright walking with rigid, jointed legs is highly efficient for traveling over flat ground. Ideally, a mobile robot would be capable of rapidly transitioning between both modes of locomotion. Using laminar jamming structures to achieve variable kinematics in robotic legs can enable flexible, continuous deformations in the unjammed state and rigid, discrete deformations in the jammed state as desired.
- **Aerodynamic control:** Two classic problems in aircraft design are stall (i.e., the loss of stability at high angles of attack due to flow separation near the wings) and aeroelastic flutter (i.e., self-excited vibration of the wings under steady flow). In the design of small-scale, fixed-wing aircraft, previous experimental and numerical investigations have shown that rigid membrane wings are highly efficient at low angles of attack, whereas

compliant membrane wings may mitigate stall at high angles of attack [95,96]. Sandwich jamming structures can be used to build thin, lightweight wings for these aircraft; furthermore, they can rapidly transition from rigid to compliant during aggressive maneuvers in order to prevent stall.

In addition, to prevent aeroelastic flutter, state-of-the-art methods typically focus on making the wings more rigid, which increases their resonant frequency and the range of velocities and payloads at which the aircraft can safely travel. However, increasing rigidity is usually associated with a significant cost and weight penalty. Incorporating laminar jamming structures into the wings can provide an intrinsic, low-cost, lightweight damping mechanism that can reduce the severity of flutter instabilities.

Other aerodynamic applications include using laminar jamming structures as tunable-stiffness elements for sails, parachutes, balloons, hang gliders, and wingsuits in order to modulate airflow over these surfaces.

- **Hydrodynamic control:** Slender swimmers (e.g., eels) can swim at different speeds by increasing their frequency of oscillation. However, with a constant body stiffness, they can achieve maximum efficiency of forward propulsion only at a certain speed. Thus, swimmers are often capable of tuning their body stiffness to ensure high efficiency across a range of speeds. Laminar jamming structures can be readily integrated into robotic swimmers as thin, lightweight, tunable-stiffness mechanisms to allow these robots to achieve the same goal.
- **Deployable structures:** The ability of human-scale laminar jamming structures to resist gravitational loading and body weight has been demonstrated in previous work [55]. This study used laminar jamming structures to build furniture that could be reconfigured before use, support the weight of a sitting adult, and rolled up and stowed away after use. An exciting opportunity for product development is to extend this idea to larger-scale structures that could be used in the home and office (e.g., reconfigurable tables, beds, floors, walls, ceilings, and walkways), as well as during emergency response operations

(e.g., laminar jamming sheets that can be rolled into a backpack in the unjammed state, and then unrolled and jammed to act as an outdoor bridge).

- **Planar robots and actuators:** Multiple types of planar robots and actuators exist, including origami-based robots and dielectric-elastomer actuators. Laminar jamming structures could be integrated into these systems to act as stiffening elements that may augment their abilities to carry loads, apply forces, and resist impacts. Furthermore, laminar jamming structures could be integrated into actuated origami sheets to selectively enable or disable certain crease patterns.
- **Rapid prototyping:** Laminar jamming sheets can be used to conform to surfaces (e.g., the human body); these formed sheets could be used as visual displays or as molds during casting processes (e.g., while casting a positive for a prosthetic limb). Furthermore, laminar jamming sheets could be combined with existing high-resolution actuated structures (e.g., actuated pin arrays) to create programmable surfaces that can then be removed and used for any prototyping operation.
- **Vibration tuning of soft actuators:** Depending on the application, vibrations in soft actuators may be desirable or undesirable. For instance, for pneumatic soft actuators that are used to apply cyclic loads at high speeds (e.g., PneuNets playing the piano [79], vibrations may be unwanted. On the other hand, for dielectric-elastomer actuators used as high-speed fluidic capacitors or acoustic elements (e.g., stretchable, transparent loudspeakers [78]), vibrations are essential for operation. As demonstrated in Chapter 3, laminar jamming structures have great potential for tuning vibrations and dynamic responses in a wide array of systems. When vibrations are not desired, laminar jamming structures may be integrated into soft actuators as tunable dampers; on the other hand, when vibrations are desired, laminar jamming structures can be used as tunable-stiffness elements to adjust resonant frequencies or mode shapes.

5.4 A Taxonomy of Jamming

As described in Chapter 1, numerous studies have been conducted on the theory and applications of granular jamming, and a smaller number of studies (including the ones presented in this thesis) have been conducted on laminar jamming. Nevertheless, few papers have directly compared the properties of different types of jamming structures (e.g., [27, 59]), and equally few have investigated what advantages can be gained by combining them (e.g., [54]).

This section presents a framework that can guide research on these less-explored topics. The framework consists of a series of diagrams that researchers in the field of jamming-based mechanisms should aim to complete and substantiate. We have made first attempts to complete these diagrams from preliminary prototypes and thought experiments; however, it is up to future studies to rigorously perform the task. Together, these diagrams can constitute a “jamming taxonomy” that can inform designers what kinds of jamming structures to build in order to meet their design requirements.

5.4.1 Selecting a Jamming Type

One question that designers may face is how to choose a type of jamming for a particular application. In this thesis, two types of jamming have been mentioned, granular jamming and laminar jamming. Consider the defining characteristic of these jamming types to be geometric dimensionality, where granular jamming is the jamming of 0-dimensional media (i.e., particles) and laminar jamming is the jamming of 2-dimensional media (i.e., sheets). It is then clear that a third type of jamming exists in between the two—that is, the jamming of 1-dimensional media (i.e., fibers). (Note that the first experimental investigation of fiber jamming in the robotics literature was recently published [97].) Thus, designers may need to choose among granular jamming, fiber jamming, and laminar jamming.

As described in this thesis, jamming structures may be used to tune stiffness, damping, kinematics, and dynamics. In the robotics literature, designers have been most interested in achieving a high jammed stiffness. Within the category of stiffness, one may consider tensile stiffness, compressive stiffness, and shear stiffness. (Note that the stiffness of an element in

bending can be derived from tensile, compressive, and shear stiffness, and the stiffness of an element in torsion can be derived from shear stiffness.) Finally, each type of stiffness is present along three different axes (i.e., x , y , and z).

With these thoughts in mind, researchers may first aim to rigorously compose a diagram like Figure 5.1. This diagram may be highly useful to designers looking to build a granular jamming structure with low or high stiffness in a particular direction; for example, if low tensile stiffness is desired (e.g., in applications that require high conformability), it is clear from the bottom-left of the diagram that granular jamming structures are preferred. In order to substantiate the predictions of such a diagram, researchers may use a combination of analytical modeling, finite-element simulation, and experimental characterization. A more formal study may aim to find the general relationship between an arbitrary stress applied to a jamming structure and the resulting strain (e.g., if the stress-strain relationship is approximated as linear, determining the components of the fourth-order stiffness tensor C_{ijkl}).

5.4.2 Selecting Jamming Types for Sandwich Structures

Another question that designers may face is how to select jamming types for the faces and cores of tunable, jamming-based sandwich structures. In this thesis, Chapter 4 rigorously explored sandwich jamming structures in which both the faces and core were laminar. However, advantages may be gained by using other jamming types within the structures. For instance, a laminar-granular sandwich jamming structure (i.e., laminar faces and a granular core) may have high tensile stiffness (from the faces) and high energy dissipation (from the core). The various combinations of jamming types are presented in Figure 5.2; for these combinations, a diagram like Figure 5.1 can be constructed.

5.4.3 Selecting Jamming Types and Geometries for Arbitrary Structures

A third question designers may face is what jamming types and geometries to select for arbitrary structures. A list of particularly compelling combinations of jamming structures and geometries, as well as their intended loading modes, is provided in Figure 5.3. Many of

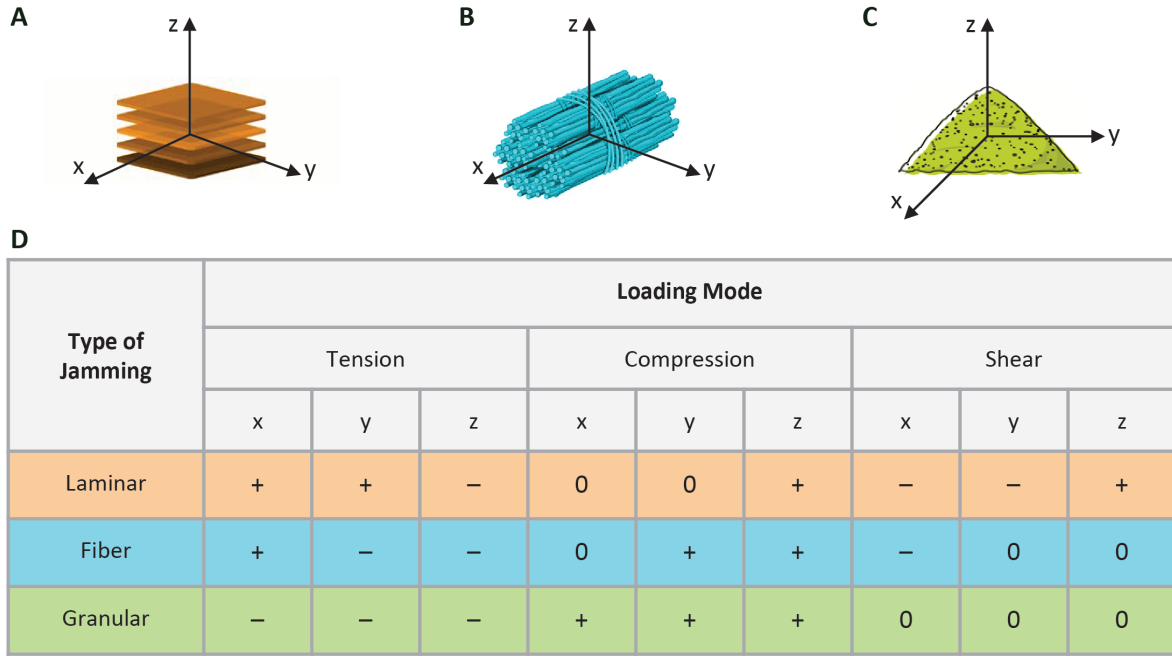


Figure 5.1: A taxonomy of jamming types. A-C) Laminar jamming, fiber jamming, and granular jamming, respectively, with the axis convention indicated. For laminar jamming, the x - and y - axes denote in-plane directions, and for fiber jamming, the x -axis denotes the fiber direction. D) Each type of jamming is qualitatively evaluated for its tensile, compressive, and shear stiffness along all three axes. A '+' indicates high stiffness, a '−' indicates low stiffness, and a '0' indicates a moderate value, relative to a specified reference. Illustrations courtesy of Buse Aktaş.

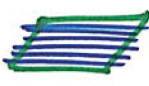
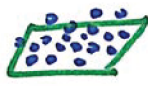
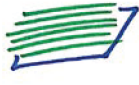
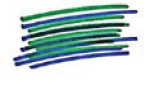
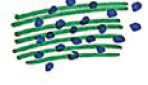
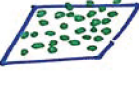
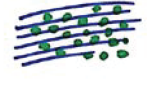
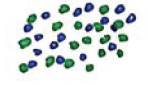
	Laminar Jamming	Fiber Jamming	Granular Jamming
Laminar Jamming			
Fiber Jamming			
Granular Jamming			

Figure 5.2: A taxonomy of jamming combinations for sandwich jamming structures. Laminar jamming, fiber jamming, and granular jamming structures may each serve as the core or face; all combinations are shown. For visual clarity, the top faces of the sandwich jamming structures are not depicted. Illustrations courtesy of Buse Aktaş.

these structures have never been presented before in the robotics literature; their intended loading modes have been estimated here via thought experiments. Researchers may aim to analytically and experimentally investigate the most promising of these structures, as well as compile a more exhaustive list that matches the major jamming types with fundamental geometries.

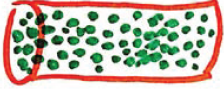
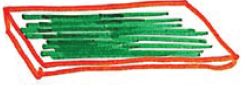
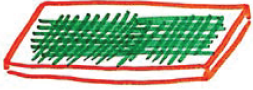
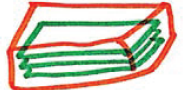
	Possible Embodiment	Description	Intended Loading
Granular Jamming		Cylinders and spheres of granular media	Compression, shear, torsion
Fiber Jamming		Laminae of uniaxially-arranged fibers	Uniaxial tension, uniaxial bending
		Laminae of biaxially-arranged fibers	Biaxial tension, biaxial bending
		Cylinders of uniaxially-arranged fibers	Uniaxial tension, shear, torsion, bending
		Helical coils of fibers	Compression, torsion
Laminar Jamming		Laminae of strips or sheets	Tension, bending
		Cylindrically-arranged or spirally-curved laminae	Tension, compression, torsion
Hybrid Jamming		Laminar-granular sandwich panels	Tension, shear, bending
		Cylinders of fibers and granular media	Tension, compression, shear, torsion

Figure 5.3: A taxonomy of jamming types and geometries for arbitrary structures. For each type of jamming, including hybrid jamming (i.e., combinations of multiple jamming types), compelling geometries are devised. For each combination of jamming type and geometry, the most suitable loading modes are qualitatively estimated and denoted. Illustrations courtesy of Buse Aktaş.

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Appendix A

Appendix to Chapter 2

A.1 Analytical Modeling

A.1.1 Governing Equations

Consider a two-layer jamming structure. Let each layer be approximated as a thin beam with a width b , height h , length L , elastic modulus E , Poisson's ratio ν , and coefficient of friction μ .

Define a coordinate system with the origin located on the left edge of the structure at the interface between the layers (Figure A.1A). Let the x -axis be horizontal (i.e., along the length of the undeformed structure), and let the y -axis be vertical (i.e., along the height of the undeformed structure).

Let the jamming structure be subject to a pressure gradient P . In this study, the jamming structure is actuated by enclosing the layers in an airtight envelope and applying a vacuum to the envelope. The pressure gradient P is equal to the vacuum pressure (i.e., the pressure in the envelope below ambient pressure). Thus, under standard atmospheric conditions, P has a maximum value of 1 *atm*.

Now let the jamming structure be loaded in the transverse direction. As the load increases, the longitudinal shear stress along the interface between the layers increases. At some regions of the interface, the longitudinal shear stress may be less than the maximum frictional stress

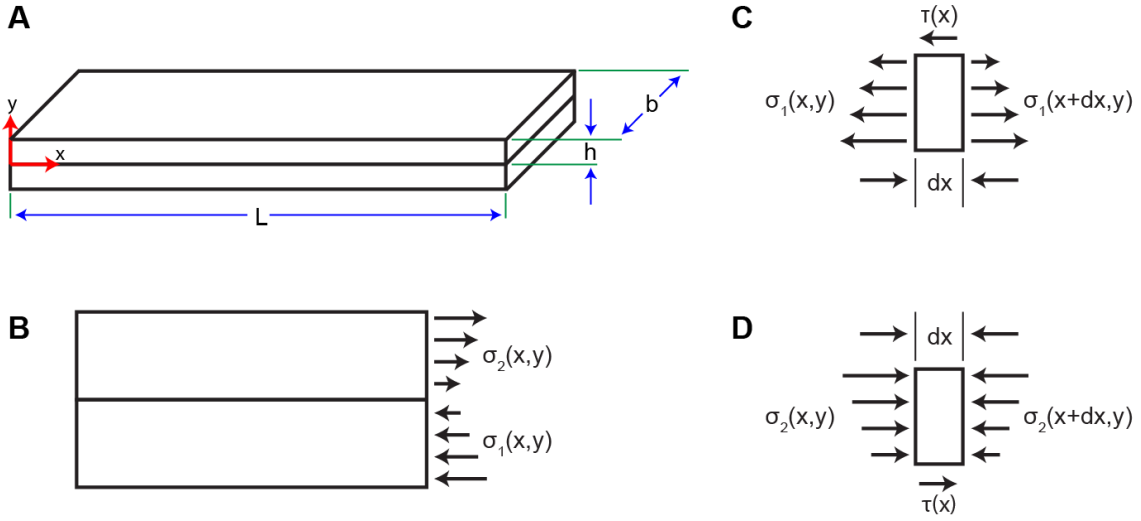


Figure A.1: Diagrams used for analytical derivation of governing equations. A) The coordinate system and dimensions for the two-layer jamming structure are defined. B) To derive the first governing equation, the resultant moment over the cross-section was computed. The resultant moment is defined as the integral of the moment of stress about the x -axis over the cross-sectional area. One possible stress distribution at a cross-section is shown. C) To derive the second governing equation, static force equilibrium of a thin section of the bottom beam was performed. Stresses were integrated over area to compute force. One possible stress distribution about a thin section is shown. D) To derive the third governing equation, static force equilibrium of a thin section of the top beam was performed.

(i.e., τ_f , which is equal to μP). These regions will remain cohesive (i.e., points that are initially coincident along the interface will remain coincident). On the other hand, at other regions of the interface, the longitudinal shear stress may equal the maximum frictional stress. These regions will slip (i.e., points that are initially coincident along the interface will move relative to each other), unless a boundary condition prevents slip from occurring.

We can write governing equations for cohesive sections of the jamming structure (i.e., sections of the jamming structure where the interface is cohesive) and slipped sections of the structure (i.e., sections of the jamming structure where the interface will slip, unless a boundary condition prevents slip from occurring).

Cohesive Sections

For cohesive sections of the jamming structure, we can write governing equations by directly using Euler-Bernoulli beam theory. (See [47] for illustrations of the fundamental assumptions of this theory.) The axial strain fields in the layers of the jamming structure are

$$\epsilon_1(x, y) = -\kappa(x) y$$

$$\epsilon_2(x, y) = -\kappa(x) y$$

where $\epsilon_1(x, y)$ and $\epsilon_2(x, y)$ are the axial strains in the bottom and top layers, respectively, and $\kappa(x)$ is the curvature along the interface.

Let us assume the layers are elastic and isotropic. The corresponding stress fields are

$$\sigma_1(x, y) = -E\kappa(x)y \tag{A.1}$$

$$\sigma_2(x, y) = -E\kappa(x)y \tag{A.2}$$

Note that when we later compare analytical results to finite element results, we substitute the plane-strain modulus $\bar{E} = \frac{E}{1-\nu^2}$ for the elastic modulus, as $b \gg h$ for the layers of the jamming structure that is investigated ([Finite Element Modeling: Two-Layer Jamming Structures](#)).

We derive the first governing equation using the relationship between the resultant moment

and the axial stress in the jamming structure (Figure A.1B). The moment-stress relation for a single beam is given by $M(x) = \int_S -\sigma(x, y) y dS$, where $\sigma(x, y)$ is the axial stress and S is the cross-section of the beam. For a two-layer jamming structure,

$$M(x) = \int_{S_1} -\sigma_1(x, y) y dS_1 + \int_{S_2} -\sigma_2(x, y) y dS_2 \quad (\text{A.3})$$

where S_1 and S_2 are the cross-sections of the bottom and top layers, respectively. Substituting equations (A.1) and (A.2),

$$M(x) = 2\kappa(x)EI \quad (\text{A.4})$$

where I is the second moment of area of a cross-section of the top layer about the interface between the layers (i.e., $\frac{bh^3}{3}$). Equation (A.4) is the only governing equation for cohesive sections of the jamming structure.

Slipped Sections

In slipped sections of the jamming structure, each layer may have a distinct neutral axis, and the vertical location of each neutral axis may vary in the horizontal direction. Thus, we can describe the axial strain fields in the bottom and top layers as

$$\epsilon_1(x, y) = -\kappa(x)y + A_1(x) \quad (\text{A.5})$$

$$\epsilon_2(x, y) = -\kappa(x)y + A_2(x) \quad (\text{A.6})$$

where $A_1(x)$ and $A_2(x)$ are axial strain components that are introduced to allow the neutral axes of the layers to be distinct.

Again assuming the layers are elastic and isotropic, the corresponding stress fields are

$$\sigma_1(x, y) = -E\kappa(x)y + EA_1(x) \quad (\text{A.7})$$

$$\sigma_2(x, y) = -E\kappa(x)y + EA_2(x) \quad (\text{A.8})$$

Substituting into equation (A.3),

$$M(x) = 2\kappa(x)EI + (A_1(x) - A_2(x))EJ \quad (\text{A.9})$$

where J is the first moment of area of a cross-section of the top layer about the interface between the layers (i.e., $\frac{bh^2}{2}$).

We derive two more equations by performing static force equilibrium. From equilibrium of thin sections of the bottom layer (Figure A.1C) and top layer (Figure A.1D), respectively,

$$\begin{aligned} -\tau(x)b dx + \int_{S_1} \sigma_1(x+dx, y) dS_1 - \int_{S_1} \sigma_1(x, y) dS_1 &= 0 \\ \tau(x)b dx + \int_{S_2} \sigma_2(x+dx, y) dS_2 - \int_{S_2} \sigma_2(x, y) dS_2 &= 0 \end{aligned}$$

where $\tau(x)$ is the shear stress exerted by the top surface of the bottom layer onto the bottom surface of the top layer. Substituting equations (A.7) and (A.8),

$$\begin{aligned} -\tau(x)b + EJ \frac{d\kappa}{dx} + ES_0 \frac{dA_1}{dx} &= 0 \\ \tau(x)b - EJ \frac{d\kappa}{dx} + ES_0 \frac{dA_2}{dx} &= 0 \end{aligned}$$

where S_0 is the cross-sectional area of a single layer (i.e., bh).

In slipped sections of the jamming structure, $\tau(x) = \tau_f$. Substituting,

$$-\tau_f b + EJ \frac{d\kappa}{dx} + ES_0 \frac{dA_1}{dx} = 0 \quad (\text{A.10})$$

$$\tau_f b - EJ \frac{d\kappa}{dx} + ES_0 \frac{dA_2}{dx} = 0 \quad (\text{A.11})$$

Since the jamming structure is loaded in the transverse direction (and not in the axial direction), the integrals of axial stress over any cross-section should be zero. From equations (A.7) and (A.8), we find that $A_1(x) + A_2(x) = 0$. Thus, equations (A.9)-(A.11) can be simplified to

$$M(x) = 2\kappa(x)EI + 2A_1(x)EJ \quad (\text{A.12})$$

$$-\tau_f b + EJ \frac{d\kappa}{dx} + ES_0 \frac{dA_1}{dx} = 0 \quad (\text{A.13})$$

Equations (A.12) and (A.13) are the two governing equations for slipped sections of the jamming structure.

A.1.2 Strain-Displacement Relations

Slipped Sections

For slipped sections of the jamming structure, it is useful to define variable $\delta_1(x)$ as the interfacial displacement for the bottom layer (i.e., the displacement of points along the top surface of the bottom layer) and variable $\delta_2(x)$ as the interfacial displacement for the top layer (i.e., the displacement of points along the bottom surface of the top layer).

From equations (A.5) and (A.6), the axial strain fields at the interface (i.e., at $y = 0$) simplify to $\epsilon_1(x) = A_1(x)$ and $\epsilon_2(x) = A_2(x)$. Thus, the interfacial displacements are related to $A_1(x)$ and $A_2(x)$ by the strain-displacement relations

$$\delta_1(x) = \int A_1(x) dx \quad (\text{A.14})$$

$$\delta_2(x) = \int A_2(x) dx \quad (\text{A.15})$$

A.1.3 Boundary Conditions

In practice, a jamming structure may be subject to one of several boundary conditions along its length (e.g., clamped, pinned, roller-supported, free). We provide clamped and free boundary conditions that will be relevant for our analysis of a cantilevered jamming structure. Additional boundary conditions can be readily derived for other physical scenarios.

Cohesive Sections

Clamped Conditions

Clamped boundary conditions at $x = a$ in cohesive sections of the jamming structure are

$$w(a) = 0 \quad (\text{A.16})$$

$$\frac{dw}{dx}(a) = 0 \quad (\text{A.17})$$

where $w(x)$ is the transverse deflection of the jamming structure at the interface (i.e., at $y = 0$).

Slipped Sections

Clamped Conditions

As in cohesive sections, clamped boundary conditions at $x = a$ in slipped sections of the jamming structure are

$$w(a) = 0 \quad (\text{A.18})$$

$$\frac{dw}{dx}(a) = 0 \quad (\text{A.19})$$

We can also formulate additional clamped boundary conditions for slipped sections that will be useful in our solution. At a clamped point, or at a boundary between slipped and cohesive sections, no interfacial displacements can occur. Thus,

$$\delta_1(a) = 0 \quad (\text{A.20})$$

$$\delta_2(a) = 0$$

Free Conditions

For a free boundary at $x = b$, we know $\sigma_1(b, y) = \sigma_2(b, y) = 0$. Substituting into equations (A.7) and (A.8) and again noting that $A_1(x) + A_2(x) = 0$, we find the boundary condition

$$\kappa(b) = 0 \quad (\text{A.21})$$

Continuity and Equilibrium

If a cohesive section and a slipped section of a jamming structure are adjacent, transverse deflections and slopes must be continuous. Symbolically, if the sections share a boundary at $x = c$,

$$w(c^-) = w(c^+) \quad (\text{A.22})$$

$$\frac{dw}{dx}(c^-) = \frac{dw}{dx}(c^+) \quad (\text{A.23})$$

In addition, axial stress is in equilibrium across the boundary (i.e., $\sigma_1(c^-, y) = \sigma_1(c^+, y)$ and $\sigma_2(c^-, y) = \sigma_2(c^+, y)$). Substituting equations (A.1), (A.2), (A.7), and (A.8) and evalu-

ating at $y = 0$, we also find the conditions

$$A_1(c^{slip}) = 0 \tag{A.24}$$

$$A_2(c^{slip}) = 0$$

where c^{slip} denotes the slipped side of the boundary.

A.1.4 Explicit Solution

In general, for a vacuumed jamming structure subject to small loads, we expect that the longitudinal shear stress along all regions of the interface will be less than the maximum frictional stress. The jamming structure will remain entirely cohesive. We call this loading regime *pre-slip*.

As we progressively increase the load, we expect that the longitudinal shear stress along some regions of the interface will equal the maximum frictional stress. Along these regions, the layers will slip (except in areas where boundary conditions prevent slip from occurring), and along other regions, the layers will remain cohesive. We call this loading regime the *transition regime*.

Finally, past a certain load, we expect that the longitudinal shear stress along all regions of the interface will equal the maximum frictional stress. The jamming structure will be entirely slipped, except at regions of the interface where boundary conditions prevent slip from occurring. We call this loading regime *full-slip*.

We now solve the boundary problem for a typical jamming structure in each of these three loading regimes. We choose to analyze a cantilevered jamming structure clamped at $x = 0$ and subject to a uniform distributed load ω ; such a case lucidly illustrates slip propagation (i.e., the gradual slip of adjacent layers along their interface), a mechanical phenomenon that jamming structures generally exhibit. (In contrast, a two-layer jamming structure in three-point bending would not exhibit slip propagation. Since longitudinal shear stress has a constant magnitude along the interface between the layers, the layers would slip along the full length of their interface at once.)

Specifically, we will provide explicit solutions for the deflection w , effective stiffness k , energy dissipated to friction E_{diss} , and effective damping d of the jamming structure. We define the effective stiffness as the incremental relationship between the distributed load and the deflection at the free end (i.e., $k = \frac{-\partial\omega}{\partial w(x=L)}$), and we define the effective damping as the incremental relationship between the dissipated energy and the deflection at the free end (i.e., $d = \frac{\partial E_{diss}}{\partial w(x=L)}$). (Note that ω is assumed to act in the negative y -direction; thus, positive ω results in negative w . The negative sign in k ensures that stiffness is positive as desired.)

Throughout the solution, we will use the small-deformation approximation $\kappa(x) \cong \frac{d^2w}{dx^2}$, where $\kappa(x)$ is the curvature of the jamming structure. This approximation allows the boundary-value problem to be explicitly solved, thus granting us deeper insight into the behavior of jamming structures. Note that when we later compare the results of the analytical model to the results of the finite element model (in which no small-deformation approximation is made), the analytical results still predict the finite element results with high accuracy ([Finite Element Modeling: Two-Layer Jamming Structures](#)).

Resultant Shear and Moment

For a jamming structure clamped at $x = 0$ with a uniform distributed load ω , the resultant shear is

$$V(x) = \omega(L - x) \tag{A.25}$$

and the resultant moment is

$$M(x) = -\frac{\omega L^2}{2} + \omega \left(Lx - \frac{x^2}{2} \right) \tag{A.26}$$

Pre-slip Regime

Deflection

During pre-slip, the jamming structure is cohesive. Thus, we can start with governing equation (A.4). Substituting equation (A.26) into equation (A.4) and solving for $\frac{d^2w}{dx^2}$,

$$\frac{d^2w}{dx^2} = -\frac{\omega L^2}{4EI} + \frac{\omega L}{2EI}x - \frac{\omega}{4EI}x^2$$

Integrating twice,

$$w(x) = -\frac{\omega L^2}{8EI}x^2 + \frac{\omega L}{12EI}x^3 - \frac{\omega}{48EI}x^4 + C_1x + C_2 \quad (\text{A.27})$$

Applying clamped boundary conditions (A.18) and (A.19) at $x = 0$,

$$w(x) = -\frac{\omega L^2}{8EI}x^2 + \frac{\omega L}{12EI}x^3 - \frac{\omega}{48EI}x^4$$

which is a standard result from Euler-Bernoulli beam theory.

Substituting the explicit expression for I provided earlier (i.e., $I = \frac{bh^3}{3}$), we find the equivalent expression

$$w(x) = -\frac{3\omega L^2}{8Ebh^3}x^2 + \frac{\omega L}{4Ebh^3}x^3 - \frac{\omega}{16Ebh^3}x^4 \quad (\text{A.28})$$

Stiffness, Dissipated Energy, and Damping

Substituting equation (A.28) into the definition of the effective stiffness of the jamming structure,

$$k = \frac{16Ebh^3}{3L^4} \quad (\text{A.29})$$

Note that the effective stiffness is constant. Thus, the coefficient of friction and the vacuum pressure have no effect on the stiffness in the pre-slip regime.

Since there is no slip in the pre-slip regime, no energy is dissipated to friction. Thus, the dissipated energy and effective damping are

$$E_{diss} = 0 \quad (\text{A.30})$$

$$d = 0 \quad (\text{A.31})$$

Transition Regime

From equation (A.25), the resultant shear is maximum at the clamped end of the jamming structure and zero at the free end; thus, longitudinal shear stress is also maximum at the clamped end and zero at the free end. Therefore, we expect that the layers would begin slipping along their interface near the clamped end, and that the slipped region would grow until reaching the free end.

Thus, in the transition regime, we can divide the jamming structure into a slipped section and a cohesive section. Let χ be the value of x where the interface transitions from slipped to cohesive. We do not know χ a priori and will solve for its value.

Slipped Section ($0 \leq x \leq \chi$)

Deflection

To calculate $w(x)$ in the slipped section of the jamming structure in the transition regime, we first find general expressions for $A_1(x)$, $\delta_1(x)$, and $w(x)$.

We begin with $A_1(x)$. Solving for $\frac{dA_1}{dx}$ in governing equation (A.13) and integrating,

$$A_1(x) = \frac{\tau_f b}{ES_0} x - \frac{J}{S_0} \frac{d^2 w}{dx^2} + C_2 \quad (\text{A.32})$$

We proceed to $\delta_1(x)$. Substituting equation (A.32) into strain-displacement relation (A.14),

$$\delta_1(x) = \frac{\tau_f b}{ES_0} \frac{x^2}{2} - \frac{J}{S_0} \frac{dw}{dx} + C_2 x + C_1 \quad (\text{A.33})$$

Finally, we proceed to $w(x)$. Substituting equation (A.26) into governing equation (A.12) and solving for $\frac{d^2 w}{dx^2}$,

$$\frac{d^2 w}{dx^2} = -\frac{\omega L^2}{4EI} + \frac{\omega}{2EI} \left(Lx - \frac{x^2}{2} \right) - \frac{J}{I} A_1(x)$$

Substituting equation (A.32),

$$\frac{d^2 w}{dx^2} \left(1 - \frac{J^2}{S_0 I} \right) = -\frac{\omega L^2}{4EI} + \frac{\omega}{2EI} \left(Lx - \frac{x^2}{2} \right) - \frac{J}{I} \left(\frac{\tau_f b}{ES_0} x + C_2 \right)$$

Integrating twice,

$$w(x) \left(1 - \frac{J^2}{S_0 I}\right) = -\frac{\omega L^2}{4EI} \frac{x^2}{2} + \frac{\omega}{2EI} \left(L \frac{x^3}{6} - \frac{x^4}{24}\right) - \frac{J}{I} \left(\frac{\tau_f b}{ES_0} \frac{x^3}{6} + C_2 \frac{x^2}{2}\right) + C_3 x + C_4 \quad (\text{A.34})$$

We can now apply clamped boundary conditions to equations (A.32)-(A.34) to explicitly solve for $w(x)$. Applying conditions (A.19) and (A.20) to equation (A.33) at $x = 0$, we find $C_1 = 0$. Next, applying conditions (A.18) and (A.19) to equation (A.34) at $x = 0$, we find $C_3 = C_4 = 0$. Finally, applying conditions (A.24) and (A.20) to equations (A.32) and (A.33), respectively, at $x = \chi$,

$$0 = \frac{\tau_f b}{ES_0} \chi - \frac{J}{S_0} \frac{d^2 w}{dx^2} \Big|_{x=\chi} + C_2 \quad (\text{A.35})$$

$$0 = \frac{\tau_f b}{ES_0} \frac{\chi^2}{2} - \frac{J}{S_0} \frac{dw}{dx} \Big|_{x=\chi} + C_2 \chi \quad (\text{A.36})$$

These equations must be consistent with the expressions for $\frac{dw}{dx} \Big|_{x=\chi}$ and $\frac{d^2 w}{dx^2} \Big|_{x=\chi}$ that can be derived from equation (A.34). Enforcing consistency and solving equations (A.35) and (A.36) for C_2 and χ , we find one trivial solution (where $\chi = 0$) and one non-trivial solution. The non-trivial solution is

$$C_2 = \frac{3(\tau_f b)^2 I}{4\omega ES_0 J} - \frac{\omega L^2 J}{16ES_0 I} - \frac{3\tau_f b L}{4ES_0} \quad (\text{A.37})$$

$$\chi = \frac{3L}{2} - \frac{3\tau_f b I}{\omega J} \quad (\text{A.38})$$

Substituting equation (A.37) into equation (A.34) and solving for $w(x)$, we find

$$w(x) = \frac{1}{1 - \frac{J^2}{S_0 I}} \left(\left(\frac{\omega(LJ)^2}{32ES_0 I^2} + \frac{3\tau_f b L J}{8ES_0 I} - \frac{3(\tau_f b)^2}{8\omega ES_0} - \frac{\omega L^2}{8EI} \right) x^2 + \left(\frac{\omega L}{12EI} - \frac{\tau_f b J}{6ES_0 I} \right) x^3 - \frac{\omega}{48EI} x^4 \right) \quad (\text{A.39})$$

As desired, equation (A.39) is the deflection in the slipped section of the jamming structure in the transition regime. Equation (A.38) provides the length of the slipped section (i.e., the length of the slipped region along the interface between the layers) as a function of the

distributed load and the maximum frictional stress.

If we substitute the explicit expressions for J , I , and τ_f provided earlier (i.e., $S_0 = bh$, $J = \frac{bh^2}{2}$, $I = \frac{bh^3}{3}$, and $\tau_f = \mu P$) into equations (A.38) and (A.39) and simplify, we find the equivalent expressions

$$\chi = \frac{3L}{2} - \frac{2\mu Pbh}{\omega} \quad (\text{A.40})$$

$$w(x) = \left(\frac{9\mu PL}{4Eh^2} - \frac{3(\mu P)^2 b}{2\omega Eh} - \frac{39\omega L^2}{32Ebh^3} \right) x^2 + \left(\frac{\omega L}{Ebh^3} - \frac{\mu P}{Eh^2} \right) x^3 - \frac{\omega}{4Ebh^3} x^4 \quad (\text{A.41})$$

This form of the expressions shows the exact functional dependence of the slipped length and the deflection on all critical design inputs (i.e., dimensions, material properties, the vacuum pressure, and the distributed load). Note that as the distributed load increases, the slipped length grows from a minimum value of zero to a maximum value of the length of the structure; the critical loads at which the slipped length begins and finishes growing are provided later in equations (A.56) and (A.57). In addition, the growth rate of the slipped length (i.e., $\frac{d\chi}{d\omega}$) scales with the vacuum pressure and the inverse square of the distributed load.

Stiffness, Dissipated Energy, and Damping

We previously defined the effective stiffness k of the jamming structure as the incremental relationship between the distributed load and the deflection at the tip. Since equation (A.41) is only valid for the slipped section of the jamming structure (i.e., for $0 \leq x \leq \chi$, where $\chi < L$), we do not yet know the deflection at the free end. Thus, we postpone the calculation of k to our subsequent investigation of the cohesive section.

Nevertheless, all the energy dissipated to friction in the transition regime arises in the slipped section, as no slip occurs in the cohesive section. Thus, we can calculate the dissipated energy E_{diss} .

We first compute $\delta_1(x)$ and $\delta_2(x)$. Substituting equation (A.37), equation (A.39), and the

result $C_1 = 0$ all into equation (A.33),

$$\delta_1(x) = \frac{1}{\left(1 - \frac{J^2}{S_0 I}\right)} \frac{\left(36(\tau_f b I)^2 - 36\omega\tau_f b L J I + 9(\omega L J)^2\right)x + \left(24\omega\tau_f b J I - 12(\omega J)^2 L\right)x^2 + 4(\omega J)^2 x^3}{48\omega E S_0 J I} \quad (\text{A.42})$$

From the earlier result $A_1(x) + A_2(x) = 0$ and the clamped boundary condition $\delta_2(0) = 0$, we find the intuitive result $\delta_2(x) = -\delta_1(x)$. We can define $\delta_r(x)$ as the relative displacement between points that were initially coincident on the interface (i.e., $\delta_1(x) - \delta_2(x)$). Thus, $\delta_r(x) = 2\delta_1(x)$.

The dissipated energy E_{diss} is the local frictional force per unit length at the interface, multiplied by the relative interfacial displacement, integrated over the length of the slipped section. Symbolically,

$$E_{diss} = \int_0^x \tau_f b \delta_r(x) dx \quad (\text{A.43})$$

Substituting $\delta_r(x)$,

$$E_{diss} = \frac{1}{\left(1 - \frac{J^2}{S_0 I}\right)} \frac{36(\tau_f b)^3 (\chi I)^2 + 4\omega(\tau_f b \chi)^2 J I (4\chi - 9L) + \omega^2 \tau_f b (\chi J)^2 (9L^2 - 8L\chi + 2\chi^2)}{48\omega E S_0 J I}$$

Substituting the explicit expressions for I , J , τ_f , and χ , we find the equivalent expression

$$E_{diss} = \frac{256(\mu P)^5 (bh)^4 - 768\omega L(\mu P)^4 (bh)^3 + 864(\omega L bh)^2 (\mu P)^3 - 432(\omega L)^3 (\mu P)^2 bh + 81(\omega L)^4 \mu P}{192\omega^3 E h^2} \quad (\text{A.44})$$

This form of the expression shows the exact functional dependence of the dissipated energy in the transition regime on all critical design inputs (i.e., dimensions, material properties, the vacuum pressure, and the distributed load).

Finally, as described earlier, we define the effective damping d as the incremental relationship between E_{diss} and the maximum deflection. From the chain rule, we know that $\frac{\partial E_{diss}}{\partial w(x=L)} = \frac{\partial E_{diss}}{\partial \omega} \frac{\partial \omega}{\partial w(x=L)}$. Simplifying, $d = -k \frac{\partial E_{diss}}{\partial \omega}$. Again, we cannot yet calculate d of the jamming structure in the transition regime, as we have had to postpone our calculation of k to the subsequent investigation of the cohesive section.

Cohesive Section ($\chi \leq x \leq L$)

Deflection

To solve for $w(x)$ in the cohesive section of the jamming structure in the transition regime, we may begin with equation (A.27). Repeating for clarity,

$$w(x) = -\frac{\omega L^2}{8EI}x^2 + \frac{\omega L}{12EI}x^3 - \frac{\omega}{48EI}x^4 + C_1x + C_2 \quad (\text{A.45})$$

Applying continuity boundary conditions (A.22) and (A.23) at $x = \chi$, we find $C_1 = 0$,

$$C_2 = \frac{1}{S_0I - J^2} \left(\frac{-9\omega L^4 J^2}{256EI} + \frac{9\tau_f b L^3 J}{32E} - \frac{27(\tau_f b L)^2 I}{32\omega E} + \frac{9(\tau_f b)^3 L I^2}{8\omega^2 E J} - \frac{9(\tau_f b)^4 I^3}{16\omega^3 E J^2} \right)$$

and

$$w(x) = -\frac{\omega L^2}{8EI}x^2 + \frac{\omega L}{12EI}x^3 - \frac{\omega}{48EI}x^4 + \frac{1}{S_0I - J^2} \left(\frac{-9\omega L^4 J^2}{256EI} + \frac{9\tau_f b L^3 J}{32E} - \frac{27(\tau_f b L)^2 I}{32\omega E} + \frac{9(\tau_f b)^3 L I^2}{8\omega^2 E J} - \frac{9(\tau_f b)^4 I^3}{16\omega^3 E J^2} \right)$$

Substituting the explicit expressions for I , J , and τ_f , we find

$$w(x) = -\frac{3\omega L^2}{8Ebh^3}x^2 + \frac{\omega L}{4Ebh^3}x^3 - \frac{\omega}{16Ebh^3}x^4 + \frac{27\mu PL^3}{16Eh^2} - \frac{(\mu P)^4 b^3 h}{\omega^3 E} + \frac{3(\mu P)^3 b^2 L}{\omega^2 E} - \frac{27(\mu P)^2 b L^2}{8\omega E h} - \frac{81\omega L^4}{256Ebh^3} \quad (\text{A.46})$$

Stiffness, Dissipated Energy, and Damping

As equation (A.46) is valid for the cohesive section of the jamming structure (i.e., for $\chi \leq x \leq L$), we now know the deflection at the free end in the transition regime and can calculate the effective stiffness k . Substituting equation (A.46) into the definition of k ,

$$k = \frac{-256\omega^4 E b h^3}{768(\mu P b h)^4 - 1536\omega L (\mu P b h)^3 + 864(\omega L \mu P b h)^2 - 129(\omega L)^4} \quad (\text{A.47})$$

Note that the effective stiffness of the jamming structure in the transition regime is a function of both the distributed load and the vacuum pressure.

We can now solve for the effective damping d of the jamming structure in the transition regime as well. Substituting equations (A.47) and (A.44) into the earlier result $d = -k \frac{\partial E_{diss}}{\partial \omega}$,

$$d = \frac{1024(\mu Pbh)^5 - 2048\omega L(\mu Pbh)^4 + 1152(\omega L)^2(\mu Pbh)^3 - 108(\omega L)^4\mu Pbh}{768(\mu Pbh)^4 - 1536\omega L(\mu Pbh)^3 + 864(\omega L\mu Pbh)^2 - 129(\omega L)^4} \quad (\text{A.48})$$

Note that the effective damping of the jamming structure in the transition regime is a function of both the distributed load and the vacuum pressure as well.

Full-slip Regime

Deflection

To solve for $w(x)$ of the jamming structure in the full-slip regime, we may begin with equation (A.32), as well as equation (A.34) after applying clamped boundary conditions (A.18) and (A.19) at $x = 0$. Providing for clarity,

$$A_1(x) = \frac{\tau_f b}{ES_0}x - \frac{J}{S_0} \frac{d^2 w}{dx^2} + C_2 \quad (\text{A.49})$$

$$w(x) \left(1 - \frac{J^2}{S_0 I}\right) = -\frac{\omega L^2}{4EI} \frac{x^2}{2} + \frac{\omega}{2EI} \left(L \frac{x^3}{6} - \frac{x^4}{24}\right) - \frac{J}{I} \left(\frac{\tau_f b}{ES_0} \frac{x^3}{6} + C_2 \frac{x^2}{2}\right) \quad (\text{A.50})$$

We cannot apply continuity boundary conditions (A.22) and (A.23), as the entire interface has slipped and the value of χ has now exceeded the length of the structure. However, we may apply free boundary condition (A.21) at $x = L$. Evaluating, we find $C_2 = \frac{-\tau_f b L}{ES_0}$. Substituting into equation (A.50) and solving for $w(x)$,

$$w(x) = \frac{1}{1 - \frac{J^2}{S_0 I}} \left(\left(\frac{\tau_f b L J}{2ES_0 I} - \frac{\omega L^2}{8EI} \right) x^2 + \left(\frac{\omega L}{12EI} - \frac{\tau_f b J}{6ES_0 I} \right) x^3 - \frac{\omega}{48EI} x^4 \right) \quad (\text{A.51})$$

Substituting the explicit expressions for I , J , and τ_f , we find the equivalent expression

$$w(x) = \left(\frac{3\mu PL}{Eh^2} - \frac{3\omega L^2}{2Ebh^3} \right) x^2 + \left(\frac{\omega L}{Ebh^3} - \frac{\mu P}{Eh^2} \right) x^3 - \frac{\omega}{4Ebh^3} x^4 \quad (\text{A.52})$$

Note that the deflection of the jamming structure in the full-slip regime is a function of the coefficient of friction and the vacuum pressure. In contrast, the deflection of a two-layer structure with a frictionless interface (or equivalently, the deflection of a two-layer structure when no vacuum is applied) is $w(x) = -\frac{3\omega L^2}{2Ebh^3} x^2 + \frac{\omega L}{Ebh^3} x^3 - \frac{\omega}{4Ebh^3} x^4$, which depends on

neither the coefficient of friction nor the vacuum pressure.

Stiffness, Dissipated Energy, and Damping

Substituting equation (A.52) into the definition of the effective stiffness of the jamming structure,

$$k = \frac{4Eb h^3}{3L^4} \quad (\text{A.53})$$

Note that the effective stiffness of the jamming structure in the full-slip regime is constant. In addition, this stiffness is equal to the effective stiffness of a two-layer structure with a frictionless interface (or equivalently, the stiffness of a two-layer structure when no vacuum is applied).

Analogous to the slipped section of the transition regime, to calculate E_{diss} , we first compute $\delta_r(x)$. We may begin with equation (A.33). Repeating for clarity,

$$\delta_1(x) = \frac{\tau_f b}{ES_0} \frac{x^2}{2} - \frac{J}{S_0} \frac{dw}{dx} + C_2 x + C_1$$

Applying clamped boundary conditions (A.19) and (A.21) at $x = 0$, we find $C_1 = 0$. Substituting equation (A.52) and the earlier result $C_2 = \frac{-\tau_f b L}{ES_0}$,

$$\delta_1(x) = \frac{1}{\left(1 - \frac{J^2}{S_0 I}\right)} \frac{(3\omega L^2 J - 12\tau_f b L I) x + (6\tau_f b I - 3\omega L J) x^2 + \omega J x^3}{12ES_0 I}$$

As before, $\delta_r(x) = 2\delta_1(x)$. Substituting into equation (A.43) with $\chi = L$,

$$E_{diss} = \frac{1}{\left(1 - \frac{J^2}{S_0 I}\right)} \frac{3\omega\tau_f b L^4 J - 16(\tau_f b)^2 L^3 I}{24ES_0 I}$$

Substituting the explicit expressions for I , J , and τ_f , we find the equivalent expression

$$E_{diss} = \frac{9\omega\mu P L^4 - 32(\mu P)^2 b h L^3}{12E h^2} \quad (\text{A.54})$$

Note that the dissipated energy in the full-slip regime is a function of both the distributed load and the vacuum pressure.

Finally, substituting equations (A.53) and (A.54) into the simplified expression for the

effective damping of the jamming structure (i.e., $d = -k \frac{\partial E_{diss}}{\partial \omega}$), we find

$$d = \mu P b h \quad (\text{A.55})$$

Note that the effective damping of the jamming structure in the full-slip regime is independent of the distributed load, but scales with the vacuum pressure. This result suggests that damping may be controlled in a real-world jamming structure over a continuum of values by forcing the structure into the full-slip regime and varying vacuum pressure as desired. This concept is investigated later for many-layer jamming structures (**Additional Concepts: Continuously-Variable Damping**).

Transition Loads Between Regimes

Let us define the *first transition load* ω_1 to be the load at which the jamming structure shifts from the pre-slip regime to the transition regime. The first transition load can be found by solving equation (A.40) for ω when $\chi = 0$. Explicitly,

$$\omega_1 = \frac{4\mu P b h}{3L} \quad (\text{A.56})$$

Let us define the *second transition load* ω_2 to be the load at which the jamming structure shifts from the transition regime to the full-slip regime. The second transition load can be found by solving equation (A.40) for ω when $\chi = L$. Explicitly,

$$\omega_2 = \frac{4\mu P b h}{L} \quad (\text{A.57})$$

Summary of Formulae

Equation (A.28) describes the deflection of the two-layer jamming structure during the pre-slip regime, and equation (A.29) describes the effective stiffness of the structure in this regime. Equations (A.30) and (A.31) describe the dissipated energy and the effective damping, which are zero.

Equations (A.40), (A.41), and (A.46) describe the deflection of the jamming structure during the transition regime (in both the slipped and cohesive sections of the structure), and

equation (A.47) describes the effective stiffness of the structure in this regime. Equations (A.44) and (A.48) describe the dissipated energy and the effective damping, respectively.

Equation (A.52) describes the deflection of the structure during the full-slip regime, and equation (A.53) describes the effective stiffness of the two-layer structure during this regime. Equations (A.54) and (A.55) describe the dissipated energy and the effective damping, respectively.

Finally, equations (A.56) and (A.57) describe the loads at which the jamming structure shifts between consecutive regimes.

Thus, we have formulated a complete model for the kinematics, stiffness, dissipated energy, and damping of a two-layer jamming structure over all major phases of deformation.

Dimensionless Forms

Through nondimensionalization, all the preceding formulae can be dramatically simplified. We can define dimensionless variables $w^* = \frac{w}{L}$, $x^* = \frac{x}{L}$, $k^* = \frac{k}{E}$, $E_{diss}^* = \frac{E_{diss}}{EL^3}$, $d^* = \frac{d}{EL^2}$, $\omega^* = \frac{\omega}{EL}$, $\mu^* = \mu$, $P^* = \frac{P}{E}$, $b^* = \frac{b}{L}$, and $h^* = \frac{h}{L}$. We can also define the composite dimensionless variables $\alpha^* = b^*h^{*3}$ and $\beta^* = \mu^*P^*b^*h^*$. Substituting these variables into the formulae, we find the following dimensionless formulae:

Pre-slip Regime

$$\begin{aligned} w^* &= \frac{-3\omega^*}{8\alpha^*}x^{*2} + \frac{\omega^*}{4\alpha^*}x^{*3} - \frac{\omega^*}{16\alpha^*}x^{*4} \\ k^* &= \frac{16\alpha^*}{3} \\ E_{diss}^* &= 0 \\ d^* &= 0 \end{aligned}$$

Transition Regime

$$k^* = \frac{-256\omega^{*4}\alpha^*}{768\beta^{*4} - 1536\omega^*\beta^{*3} + 864\omega^{*2}\beta^{*2} - 129\omega^{*4}}$$

$$E_{diss}^* = \frac{256\beta^{*5} - 768\omega^*\beta^{*4} + 864\omega^{*2}\beta^{*3} - 432\omega^{*3}\beta^{*2} + 81\omega^{*4}\beta^*}{192\omega^{*3}\alpha^*}$$

$$d^* = \frac{1024\beta^{*5} - 2048\omega^*\beta^{*4} + 1152\omega^{*2}\beta^{*3} - 108\omega^{*4}\beta^*}{768\beta^{*4} - 1536\omega^*\beta^{*3} + 864\omega^{*2}\beta^{*2} - 129\omega^{*4}}$$

Slipped Section

$$\chi^* = \frac{3}{2} - \frac{2\beta^*}{\omega^*}$$

$$w^* = \left(\frac{9\beta^*}{4\alpha^*} - \frac{3\beta^{*2}}{2\omega^*\alpha^*} - \frac{39\omega^*}{32\alpha^*} \right) x^{*2} + \left(\frac{\omega^*}{\alpha^*} - \frac{\beta^*}{\alpha^*} \right) x^{*3} - \frac{\omega^*}{4\alpha^*} x^{*4}$$

Cohesive Section

$$w^* = \frac{-3\omega^*}{8\alpha^*} x^{*2} + \frac{\omega^*}{4\alpha^*} x^{*3} - \frac{\omega^*}{16\alpha^*} x^{*4} + \frac{27\beta^*}{16\alpha^*} - \frac{\beta^{*4}}{\omega^3\alpha^*} + \frac{3\beta^{*3}}{\omega^{*2}\alpha^*} - \frac{27\beta^{*2}}{8\omega^*\alpha^*} - \frac{81\omega^*}{256\alpha^*}$$

Full-slip Regime

$$w^* = \left(\frac{3\beta^*}{\alpha^*} - \frac{3\omega^*}{2\alpha^*} \right) x^{*2} + \left(\frac{\omega^*}{\alpha^*} - \frac{\beta^*}{\alpha^*} \right) x^{*3} - \frac{\omega^*}{4\alpha^*} x^{*4}$$

$$k^* = \frac{4\alpha^*}{3}$$

$$E_{diss}^* = \frac{9\omega^*\beta^* - 32\beta^{*2}}{12\alpha^*}$$

$$d^* = \beta^*$$

Transition Loads

$$\omega_1^* = \frac{4\beta^*}{3}$$

$$\omega_2^* = 4\beta^*$$

Note that dimensionless deflections are found to depend on at most 4 dimensionless parameters (i.e., ω^* , x^* , α^* , and β^*); dimensionless stiffnesses and dissipated energies depend on at most 3 parameters (i.e., ω^* , α^* , and β^*); dimensionless damping values depend on at most 2 parameters (i.e., ω^* and β^*); and dimensionless transition loads depend on just 1 parameter (i.e., β^*).

A.1.5 Case Study

The analytical model was evaluated for an example two-layer jamming structure. Each layer had dimensions $b = 50 \text{ mm}$, $h = 0.1 \text{ mm}$, and $L = 250 \text{ mm}$, as well as a Poisson's ratio $\nu = 0.156$ and coefficient of friction $\mu = 0.65$. These dimensions and material properties coincided with those of the real-life jamming structures examined later during experimental characterization ([Experimental Characterization](#)).

If a two-layer jamming structure with the above properties consisted of compliant material, it would not slip until the structure exhibited exceptionally large deflections. Thus, the elastic modulus E was set to 6 TPa in order to illustrate slip over a more reasonable range of deflection. In addition, as described earlier, the plane-strain modulus $\bar{E} = \frac{E}{1-\nu^2}$ was substituted for the elastic modulus in the analytical formulae, as $b \gg h$.

A vacuum pressure $P = 101 \text{ kPa}$ was imposed, and a uniform distributed load $\omega = 7 \frac{\text{N}}{\text{m}}$ was applied over 100 equal increments. The elastica (i.e., shape), the deflection at the free end of the jamming structure, and the dissipated energy were computed for each load increment.

A.1.6 Curvature Reversal

For a typical cantilever beam under a uniform distributed load, the curvature of the beam maintains a consistent sign. However, for a two-layer jamming structure, the analytical model predicts that the curvature reverses (i.e., changes sign) along its length at moderate loads and higher. Curvature reversal can be seen on close inspection of the elastica in the case study ([Figure A.2B](#)).

The analytical model in the transition regime may provide a first explanation of this counterintuitive phenomenon. Because the net force on any cross section is zero, $A_1(x) + A_2(x) = 0$ for all x . Furthermore, because the jamming structure is cohesive for $\chi \leq x < L$, interfacial displacements must be zero at $x = \chi$. Thus, positive values of $A_1(x)$ within the slipped section of the jamming structure (i.e., $0 \leq x \leq \chi$) must be balanced by negative values of $A_1(x)$ elsewhere in the slipped section; likewise, positive values of $A_2(x)$ must be balanced by negative values of $A_2(x)$. Equations [\(A.5\)](#) and [\(A.6\)](#) imply a similar (but not identical)

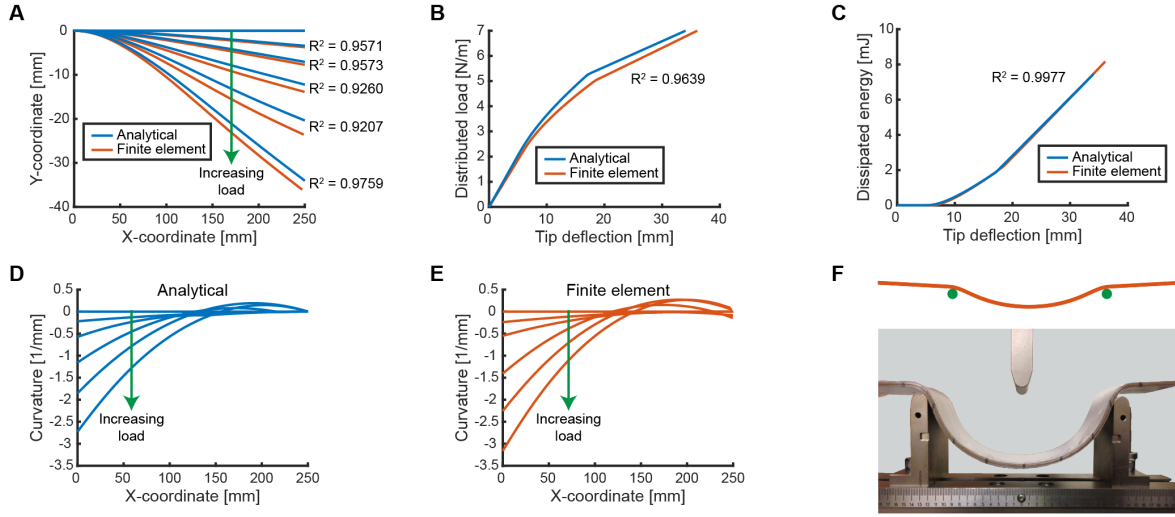


Figure A.2: *Finite element evaluation of analytical model. Analytical and finite element models were constructed of a two-layer jamming structure in cantilever bending subject to a uniform distributed load. The models had identical dimensions, material properties, boundary conditions, and loads. The analytical model predicted finite element results with high accuracy in all cases. A) Elastica are compared for six equal load increments from zero load to the maximum load. B) Load-versus-deflection curves are compared. Recall that the effective stiffness k is equal to the slope. C) Dissipated-energy-versus-deflection curves are compared. Recall that the effective damping b is equal to the slope. D) Curvatures are shown for the elastica of the two-layer analytical model. Note that the curvature crosses zero (i.e., reverses sign) for moderate loads and above. E) Curvatures are shown for the two-layer finite element model. The curvature profiles are predicted closely by the analytical model. F) Curvature reversal was also observed for finite element models (Top) and experimental samples (Bottom) of many-layer jamming structures in three-point bending. Sharp curvature reversal can be seen near the supports.*

relationship for $\kappa(x)$.

Curvature reversal was corroborated in detail by finite element models of two-layer jamming structures, as well as finite element models and experimental observations of many-layer jamming structures ([Finite Element Modeling: Two-Layer Jamming Structures](#)).

A.1.7 Extending the Model

The analytical modeling procedure for two-layer jamming structures can be adapted to solve for the deflection of jamming structures with arbitrary numbers of layers and arbitrary boundary conditions.

Arbitrary Numbers of Layers

Three important results may be simply derived for many-layer jamming structures. First, the elastica of a vacuumed jamming structure during the pre-slip regime can be determined by approximating the structure as a cohesive thin beam and directly using Euler-Bernoulli beam theory to calculate deflection.

Second, as cited in the main text, the bending stiffness of a vacuumed many-layer jamming structure during the pre-slip regime is a factor of n^2 greater than the stiffness when no vacuum is applied, where n is the number of layers [34]. This result can be derived from second area moments of inertia. When a vacuumed jamming structure is in the pre-slip regime, the structure is cohesive, and the second area moment of inertia of the jamming structure is given by $I = \frac{b(nh)^3}{12} = n^3 \frac{bh^3}{12}$. When no vacuum is applied to a jamming structure, the layers are decoupled, and $I = n \frac{bh^3}{12}$. Since bending stiffness is proportional to I , the stiffness in the former case is a factor of n^2 greater than that in the latter case.

Third, the first transition load (i.e., the load at which a jamming structure shifts from the pre-slip regime to the transition regime) for many layer-jamming structures is given by $V_{max} = \frac{2\mu PA}{3}$, where V_{max} is the maximum resultant shear at any cross-section of the beam, which is proportional to the applied load; and A is the total cross-sectional area (i.e., nbh) [65]. This result can be derived from the definition of slip. Slip occurs when the maximum

longitudinal shear stress at an interface equals the maximum possible shear stress. During the pre-slip regime, the maximum longitudinal shear stress occurs at the innermost interface and is given by the well-known formula $\tau_{max} = \frac{3V_{max}}{2A}$. Furthermore, the maximum possible shear stress is μP . Equating the two expressions and solving for V_{max} , we see $V_{max} = \frac{2\mu PA}{3}$.

Despite the simplicity of deriving the previous three results, solving for the deformation of a many-layer jamming structure during the transition regime and full-slip regime is a far greater challenge. This paper has provided detailed methods for solving for the deformation of a two-layer jamming structure during these regimes; these methods may be extended to solve the many-layer problem as well.

For a many-layer jamming structure, strain distributions can again be defined for each layer as the superposition of a linear strain term and a unique axial strain term. The moment-stress relation can be used to derive a first governing equation. Static force equilibrium can then be performed on thin sections of each layer to derive subsequent governing equations. For the outermost layers, these equations would be similar to equations (A.10) and (A.11) for the two-layer jamming structure; for inner layers, shear stress would act on both the top and bottom surfaces of the thin section, producing a second shear stress term.

Whereas slip propagates along one dimension (i.e., along the x -axis, as defined in Figure A.1A) for a two-layer jamming structure, slip would propagate along two dimensions (i.e., along the x - and y -axes) for a many-layer jamming structure. Shear stress varies through the thickness of the structure, and slip would occur along distinct interfaces at disparate loads. Thus, a unique χ variable would be required for each interface. Moreover, boundary conditions (A.24) and (A.20) would only be valid at $x = \chi$ if χ corresponded to an interface located along the x -axis; for other interfaces, alternative boundary conditions would need to be formulated. For example, continuity of incremental interfacial displacements and axial strains may be enforced.

Although the process of solving for the deflection of a many-layer jamming structure is straightforward, the solution itself may be algebraically taxing. Furthermore, the analytical solution would only be valid for small deflections. Thus, numerical solutions (e.g., finite

element analysis) may be far more convenient.

Arbitrary Boundary Conditions

For arbitrary boundary conditions, the direction of shear stress and frictional stress may change along the length of an interface. Thus, the signs in the governing equations based on static force equilibrium may vary throughout the jamming structure. Furthermore, each interface may consist of multiple slipped and cohesive regions. More than one χ variable would be necessary for each interface, along with continuity boundary conditions between adjacent slipped and cohesive sections of the structure.

A.2 Finite Element Modeling

All finite element models were constructed using finite element simulation software (ABAQUS 6.14r2, Dassault Systèmes, Villacoublay, France). Analysis of simulation results was performed using numerical computing software (MATLAB 2017a, MathWorks, Natick, MA).

A.2.1 Two-Layer Jamming Structures

A finite element model was constructed for a two-layer jamming structure. Each layer was approximated as a 2D plane-strain structure, and the jamming structure had dimensions, material properties, vacuum pressure, and distributed load equal to those specified in the case study for the analytical model of a two-layer jamming structure ([Analytical Modeling: Case Study](#)).

Boundary conditions and loads were also identical to those used in the case study. First, pressure (equal to the vacuum pressure) was applied to all outer surfaces of the jamming structure; then, the uniform distributed load was applied as a ramp over 100 equal increments. Large-deformation analysis was enabled. The interface between the two layers was chosen to be a contact surface with a penalty friction formulation. To mitigate undesired simulation of elastic slip, the slip tolerance was set to $5 * 10^{-5}$. A uniform mesh was used that consisted of square four-node bilinear plane-strain quadrilateral elements with reduced integration (CPE4R).

Each layer had two elements across its thickness. A mesh refinement study was conducted later for many-layer jamming structures to ensure that a finer mesh was not required (**Finite Element Modeling: Stiffness of Many-Layer Jamming Structures**). The elastica, deflection at the free end, and dissipated energy were extracted at each load increment.

The analytical model predicted finite element results for a two-layer jamming structure with high accuracy (Figure A.2A-C). Elastica were predicted with coefficients of determination (R^2) between 0.9207 and 0.9759. Furthermore, the load-versus-deflection curve of the structure was predicted with $R^2 = 0.9639$, and the dissipated-energy-versus-deflection curve was predicted with $R^2 = 0.9977$. Note that finite element models of many-layer jamming structures were later found to predict experimental results with exceptional accuracy; thus, the analytical model was deemed predictive of real-world jamming structures as well.

The curvature reversal phenomenon predicted by the analytical model (**Analytical Modeling: Curvature Reversal**) was also corroborated by the finite element results (Figure A.2D-E). For the analytical model, curvatures were computed for each of the analytical elastica in Figure A.2A using appropriate first and second derivatives of the formulae for deflection (**Analytical Modeling: Summary of Formulae**). For the finite element model, fourth-order best-fit polynomials were first determined for each of the finite element elastica in Figure A.2A. Curvatures were then computed using appropriate first and second derivatives of the best-fit polynomials. The analytical curvature profiles (Figure A.2D) were visually predictive of the finite element curvature profiles (Figure A.2E), including the x -coordinates at which curvature reversal occurred.

Curvature reversal was also observed later for both finite element models and experimental samples of many-layer jamming structures in three-point bending (Figure A.2F).

A.2.2 Stiffness of Many-Layer Jamming Structures

Finite element models of many-layer jamming structures were constructed according to the same process used for two-layer jamming structures; however, the structures were loaded in three-point bending. Rollers (i.e., zero-vertical-displacement boundary conditions) were applied

to two points on the bottom surface, 60 *mm* from either side; the location of these virtual rollers coincided with the location of the rollers used later during experimental characterization. In addition, to stabilize the finite element model, a zero-horizontal-displacement boundary condition was applied at the center of the top surface of the structure.

After applying pressure to all outer surfaces of the jamming structure, a concentrated transverse displacement was applied to the midpoint of the top surface. The displacement had a minimum value of 0 *mm* and a maximum value of 8 *mm* and was applied as a ramp over 100 equal increments. The displacement range coincided with the range used later during experimental characterization. The interface between each pair of adjacent layers was chosen to be a contact surface with a penalty friction formulation. As with the two-layer jamming structures, a uniform mesh of square elements was used, and each layer had two elements across its thickness.

A mesh refinement study was conducted to ensure that a finer mesh was not required. The study was performed for a twenty-layer jamming structure with vacuum pressure $P = 71.1 \text{ kPa}$ and coefficient of friction $\mu = 0.65$. The number of elements across the thickness of each layer was varied between one and four (i.e., the areal density of elements was varied by a factor of sixteen). The one-element simulation did not converge; however, the two-, three-, and four-element simulations converged successfully. In each of the converged simulations, the transverse load and maximum deflection were extracted at each displacement increment. The converged simulations produced force-versus-maximum-deflection curves that were nearly indistinguishable (Figure A.3A). The mean force difference between the two- and three-element simulations was 0.050 *N* (0.54% of the range of the two-element simulation), and the mean difference between the two- and four-element simulations was 0.073 *N* (0.78% of the range of the two-element simulation). Thus, it was deemed sufficiently accurate to mesh each layer with just two elements across its thickness.

The three major design inputs (i.e., the number of layers n , vacuum pressure P , and coefficient of friction μ) were then systematically varied. The quantity n was varied from 5 to 20 in increments of 5 (with $P = 71.1 \text{ kPa}$ and $\mu = 0.65$); P was varied from 23.7 *kPa* to

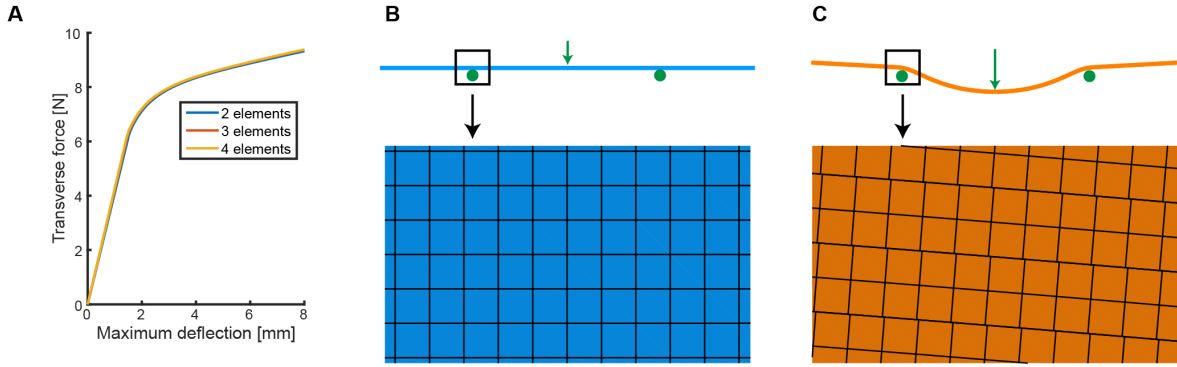


Figure A.3: *Finite element mesh refinement study and slip visualization. A) A mesh refinement study was performed for a twenty-layer finite model in three-point bending. A uniform mesh of square elements was used, and the number of elements across the thickness of each layer was varied between two and four. The resulting force-versus-deflection curves were nearly indistinguishable; thus, two elements across the thickness was sufficiently accurate. B) The many-layer finite element models could illustrate slip between adjacent layers. Slip for a typical twenty-layer model in three-point bending is shown here. During the pre-slip regime, nodes along adjacent interfaces were coincident. C) During the full-slip regime, nodes that were initially coincident moved relative to each other.*

71.1 kPa in increments of 23.7 kPa (with $n = 20$ and $\mu = 0.65$); and μ was varied from 0.25 to 1 in increments of 0.25 (with $n = 20$ and $P = 71.1$ kPa). For each set of design inputs, the transverse force and maximum deflection were extracted at each displacement increment. Recall that the effective stiffness k is simply the slope of the force-versus-maximum-deflection curve.

Aside from providing information about the macroscopic deformation of many-layer jamming structures, the finite element models also illustrated the microscopic phenomenon of the slipping of adjacent layers along their interface at high loads (Figure A.3B-C).

A.2.3 Damping of Many-Layer Jamming Structures

In laminar jamming structures, the layers are coupled via dry friction. The relevant damping phenomenon is Coulomb damping, in which the damping force is independent of the rate of deformation (as opposed to viscous damping, in which the damping force is rate-dependent). Even when jamming structures are loaded quasi-statically, energy is still dissipated. Thus, finite element models of jamming structures subject to static loading are sufficient to characterize damping, and dynamic simulations are not required.

If interfacial velocities (i.e., the velocities at which adjacent layers slip) were high, the damping force could theoretically become rate-dependent. However, from the second term of equation (A.33) in **Analytical Modeling: Explicit Solution**, interfacial displacements are observed to scale with $\frac{h}{L}$ times the transverse deflection, where h is the thickness of a layer and L is the length. In the jamming structures analyzed in the paper, h is smaller than L by four orders of magnitude; thus, interfacial velocities are negligible unless transverse velocities are exceptionally high.

The finite element models to analyze damping were built according to the same process as the finite element models to analyze stiffness. However, after the maximum input displacement of 8 mm was applied, the transverse load was reduced to 0 N over 100 equal increments. The force-versus-maximum-deflection curves then illustrated hysteresis, and the area under the curves depicted the energy dissipated over the loading cycle.

Recall that the effective damping d is simply the dissipated energy per unit deflection. The quantity d was not explicitly calculated, but can easily be determined. For each point on the force-versus-maximum-deflection curve, an elastic unloading line can be drawn (with a slope equal to that of the pre-slip loading line), and the area under the resulting curve can be computed. This area is the dissipated energy at that particular deflection. After performing this procedure for all points, the dissipated energy can then be plotted against deflection. The value of d is the slope of this curve.

A.2.4 Functional Dependencies

The finite element simulations for the many-layer jamming structures were rerun over an extended displacement range (from 0 mm to 16 mm over 400 equal increments) to ensure that all structures entered the full-slip regime, allowing accurate measurement of full-slip stiffness and damping. Furthermore, the simulations were executed for larger sets of the design inputs to provide more data points for determining functional dependence. The numbers of layers examined were 2, 5, 7, 10, 12, 13, 15, 17, 18, and 20; the vacuum pressures were 0.34, 11.9, 23.7, 35.6, 47.4, 59.3, 71.1, 83.0, 94.8, and 101.1 kPa; and the coefficients of

friction were 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.75, and 0.8.

For each simulation, best-fit lines were fit to the first 1% and the last 1% of the force-versus-maximum-deflection curve. The slope of the former line approximated the effective stiffness k during pre-slip, whereas the slope of the latter line approximated k during full-slip. A best-fit line was then fit to the last 1% of the dissipated-energy-versus-maximum-deflection curve. The slope of this line approximated the effective damping d during full-slip. (Recall that d during pre-slip is simply 0).

For each design input (e.g., number of layers), each performance metric (e.g., pre-slip stiffness) was plotted against the values of the design input (e.g., 2 layers, 5 layers, 7 layers, etc.). Based on the formulae derived in the analytical model for two-layer jamming structures, it was hypothesized that the performance metrics for many-layer jamming structures had polynomial dependence on the design inputs. Thus, best-fit polynomials were fit to each plot; however, the appropriate order for each polynomial needed to be determined.

Best-fit polynomials from zero- to fourth-order were tested on each plot, and the root-mean-square error e_{rms} was computed for each polynomial. From physical reasoning, pre-slip stiffness should be unaffected by the coefficient of friction μ and the vacuum pressure P , as jamming structures are cohesive in pre-slip; thus, the pre-slip stiffness should have zero-order dependence on μ and P . When zero-order polynomials (i.e., flat lines) were fit to the pre-slip stiffness plots for μ and P , it was found that $e_{rms} \cong 0.0070 \frac{N}{mm}$. This value quantified numerical noise in the finite element simulations and was used as the cutoff for determining the appropriate order of the best-fit polynomial for the other stiffness plots. Specifically, for a given pre-slip or full-slip stiffness plot, the lowest-order best-fit polynomial for which $e_{rms} \leq 0.0070 \frac{N}{mm}$ was determined to be the appropriate polynomial.

For the full-slip damping plots, dimensional analysis suggested that the e_{rms} threshold should be multiplied by a characteristic length in order to exhibit the correct units (i.e., $[N]$). As the layers in a jamming structure slip in the direction of their length, the length $L = 250 \text{ mm}$ was chosen as the characteristic length, and $e_{rms} \cong 0.0070 \frac{N}{mm} * 250 \text{ mm} = 1.8 \text{ N}$ was used as the cutoff for determining the appropriate order of the best-fit polynomial for the damping

Table A.1: *Functional dependence of performance metrics on design inputs for many-layer jamming structures. Regression analysis was used to determine the functional dependence of stiffness and damping on the number of layers, vacuum pressure, and coefficient of friction. The relationships between the parameters were well-described by best-fit polynomial functions of the specified order. Root-mean-square (RMS) error is provided for each polynomial relationship.*

Design Input	Performance Metric					
	Pre-Slip Stiffness		Full-Slip Stiffness		Full-Slip Damping	
	Polynomial Order	RMS Error $[\frac{N}{mm}]$	Polynomial Order	RMS Error $[\frac{N}{mm}]$	Polynomial Order	RMS Error $[\frac{N}{mm}]$
Number of Layers (n)	3	0.0001	1	0.0006	1	0.0238
Vacuum Pressure (P)	0	0.0065	1	0.0028	1	0.1326
Coefficient of Friction (μ)	0	0.0072	2	0.0020	1	0.0788

plots.

The polynomial orders were then tabulated to assess functional dependencies (Table A.1). As expected, pre-slip and full-slip stiffness scaled with n^3 and n , respectively, where n is the number of layers. Full-slip stiffness also scaled with P and μ^2 . In contrast, the analytical model predicted that the full-slip stiffness of a two-layer jamming structure was independent of P and μ . The dependence of full-slip stiffness on these quantities in the finite element model is likely a result of contact pressure distributions arising from the concentrated load and roller supports.

Full-slip damping scaled with n , P , and μ . Damping should scale with the number of interfaces, which in turn scales with n ; furthermore, damping should scale with the frictional stress at the interfaces, which again is equal to μP everywhere during full-slip. Thus, these scaling relationships are also physically reasonable. Note that the scaling of full-slip damping with P and μ was also predicted by the analytical model for a two-layer jamming structure.

For practical applications, one final functional dependence is critical: the dependence of the first transition load for a many-layer jamming structure (i.e., the load at which the

jamming structure moves from the pre-slip regime to the transition regime) on the design inputs. However, finite element analysis was not necessary to determine this dependence; as described earlier, the first transition load can be accurately predicted by Euler-Bernoulli beam theory and scales with n , P , and μ ([Analytical Modeling: Extending the Model](#)). Note that the scaling of this load with P and μ was predicted by the analytical model for a two-layer jamming structure as well.

A.2.5 Limiting Behavior

For practical applications, the limiting behavior of jamming structures may be useful to examine. Consider an application in which the bending stiffness ratio between the jammed and unjammed states must be maximized (e.g., for a splint that must gently conform to the shape of a limb and then stiffen to immobilize a joint). This goal can be accomplished by constructing the layers out of exceptionally thin material (e.g., metal foil) and stacking as many layers as possible within the total allowable height H . When the structure is unjammed, the stiffness will be negligible, as the layers are thin and flexible. When the structure is jammed, the stiffness will be equal to that of a cohesive metal structure of height H .

Nevertheless, such a configuration may have adverse consequences. If the structure is jammed and unintentionally forced into the full-slip regime (e.g., upon a collision), the structure will then exhibit a stiffness approximately equal to its unjammed stiffness; because the unjammed stiffness is negligible, the structure will yield catastrophically if the load is maintained. Obtaining accurate predictions of the load-deformation curve during the transition regime and full-slip may help designers avoid such consequences. Unfortunately, many-layer finite element simulations can be computationally expensive when the number of layers (and in turn, the number of contact interactions) are particularly large.

One solution would be to approximate the system by the limiting case in which the layers of the jamming structure are infinitesimally thin, and an infinite number of layers are stacked within the height H . In other words, the structure is approximated as a continuum. The structure may then be modeled as a single crystal with a single slip system, with slip planes

parallel to the xz -plane and the slip direction parallel to the x -axis. The structure may then be computationally modeled using existing finite element packages for crystal plasticity (e.g., [98]).

A.2.6 Variable Kinematics

The variable kinematics system was modeled as four parts: one rubber substrate and three jamming structures adhered to the bottom. The substrate was approximated as a 2D plane-strain structure with in-plane dimensions of $150\text{ mm} \times 20\text{ mm}$. Each jamming structure represented a twenty-layer jamming structure, but was approximated as a homogeneous 2D plane-strain structure. The in-plane dimensions of each jamming structure were $49.33\text{ mm} \times 20\text{ mm}$, and the thickness was equal to the total thickness of twenty layers of paper (i.e., 2 mm). Adjacent jamming structures were separated by 1 mm gaps.

Both the substrate and the jamming structures were approximated as elastic. The substrate in subsequent experimental validation was cast from high-stiffness PDMS rubber (Sylgard 184, Dow Corning, Midland, MI). To accurately model this substrate in finite element simulations, the stress-strain curve reported in the literature for the PDMS rubber in uniaxial tension was digitally traced over small deformations [99]. The elastic modulus was determined by computing the slope of the curve, and this elastic modulus (i.e., 19.1 MPa) was assigned to the substrate in the finite element model.

Each jamming structure was assigned an elastic modulus in its vacuum-on state and its vacuum-off state. In the vacuum-on state, the elastic modulus equaled that of paper (6 GPa); in the vacuum-off state, the elastic modulus was reduced by a factor of n^2 (15 MPa). The substrate and jamming structures were assigned a Poisson's ratio of 0.49 and 0.156, respectively. Finally, as described in the main text, the thickness of the rubber substrate was chosen such that the bending stiffness of the substrate (k_{sub}) was the geometric mean of the bending stiffness of the jamming structures in the vacuum-off state (k_{jam}^{nv}) and the bending stiffness in the vacuum-on state (k_{jam}^v). Because the jamming structures were intended to deform exclusively in the pre-slip regime, standard Euler-Bernoulli beam theory could be used

to approximate bending stiffness simply as EI , where E is the elastic modulus and I is the second area moment of inertia. Using this approximation, the desired thickness of the rubber substrate was 5.0 mm .

One end of the rubber substrate was fixed. To approximate the effect of cable actuation, a pure moment load was applied to a point on the free end. Two simulations were executed: one where the jamming structures were assigned their vacuum-off elastic modulus, and another where they were assigned their vacuum-on modulus. The magnitude of the moment loads were chosen such that the free end of the rubber substrate would nearly contact the fixed end at maximum load; the vacuum-off simulation had a maximum load of 350 $N * mm$, and the vacuum-on simulation had a maximum load of 1 $N * mm$. The loads were applied as ramps over 400 equal increments, and large deformation analysis was enabled. A uniform mesh was used that consisted of square four-node bilinear plane-strain quadrilateral hybrid elements with reduced integration (CPE4RH). Four elements were used across the thickness of the structure.

For both the vacuum-on and vacuum-off cases, the shape of the variable kinematics system was visualized at each load increment. In addition, the coordinates of the nodes along the ventral surface of the system (i.e., the longitudinal surface with the smaller radius of curvature when the system was actuated) were extracted at each load increment. The exact local curvatures were then calculated along the surface using appropriate first and second derivatives of the coordinates.

A.3 Experimental Characterization

A.3.1 Fabrication Process

The many-layer jamming structures used in experimental characterization consisted of three parts: strips of copy paper (HP Ultra White Multipurpose Copy Paper), an envelope of thermoplastic polyurethane with a thickness of 0.038 mm (American Polyfilm, Inc., Branford, CT), and thermoplastic polyurethane tubing with an outer diameter of 3 mm (Eldon James

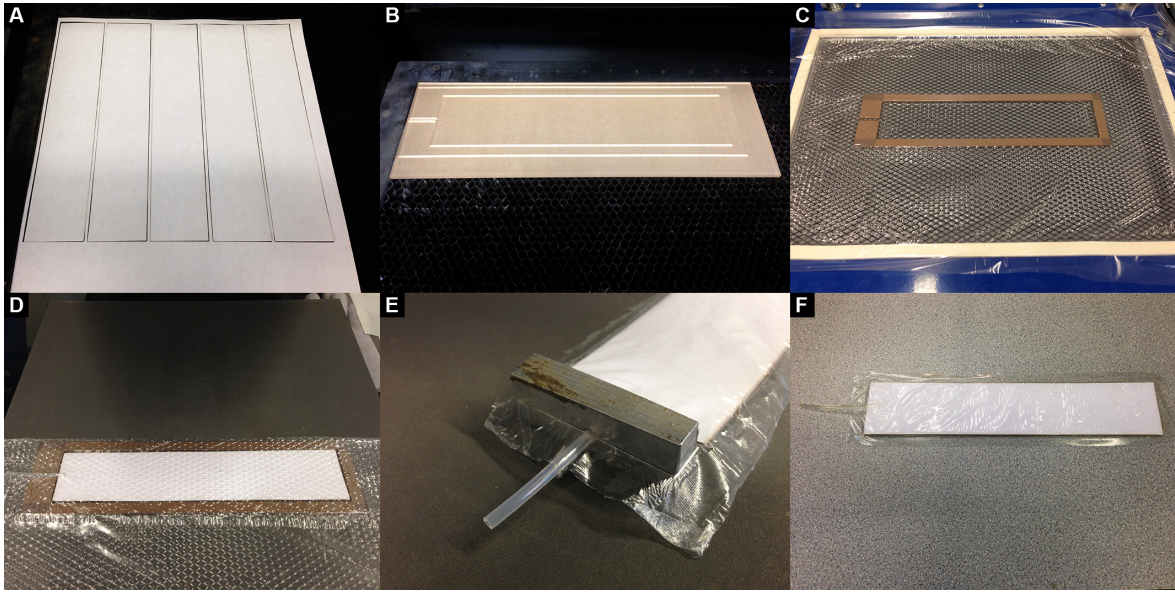


Figure A.4: *Fabrication process for real-world jamming structures. The fabrication process is illustrated for a typical sample of a many-layer jamming structure. A) A sheet of copy paper was placed on the bed of a laser cutter, and strips were cut along the machine direction. B) Acrylic was placed on the bed of the laser cutter, and a frame was cut. C) The frame was coated in PTFE tape and positioned on the bed of a vacuum former. A thin sheet of TPU was formed to the frame. D) The frame and TPU sheet were positioned on the bed of a heat press. The strips of paper and a segment of TPU tubing were placed inside, and the TPU sheet was folded to cover the contents. E) Aluminum blocks with a circular channel were arranged on the top and bottom of the end of the jamming structure, sandwiching the TPU tubing between the two sides of the TPU envelope. F) Fabrication of the sample was completed.*

Corp., Denver, CO).

The fabrication process for the laminar jamming samples consisted of five major steps (Figure A.4). First, the strips of copy paper were manufactured. Sheets of copy paper were placed on a laser cutter (VLS4.60, Universal Laser Systems, Inc., Scottsdale, AZ), and strips were cut along the machine direction of the paper (i.e., the long axis) (Figure A.4A).

Next, the thermoplastic polyurethane (TPU) envelope was created. A frame was cut on the laser cutter from acrylic plastic (Figure A.4B); this frame defined the region of the TPU sheet that would be sealed in a later step. Since the TPU sheet was intended to form an envelope around the paper strips and TPU tubing, the shape of the frame comprised a close perimeter around these contents. Furthermore, since the frame would be in contact with hot elements in subsequent steps, it was coated with polytetrafluoroethylene (PTFE) tape to

prevent adherence.

The TPU sheet was then formed to the acrylic frame on a vacuum former (Formech 300XQ, Formech International Limited, Hertfordshire, United Kingdom) to create a pocket in which to place the paper and tubing (Figure A.4C). After placing the paper and tubing into the pocket, the sheet of TPU was folded once upon itself to enclose the pocket (Figure A.4D). The TPU sheet was then covered temporarily with a PTFE sheet and heat-sealed using a one-sided heat press (Powerpress, Fancierstudio, Hayward, CA) at 100 °C. Since the heat conduction to the TPU was greatest through the acrylic frame, only the region of the TPU sheet in contact with the frame was sealed, forming an envelope.

To prevent leakage of air into the envelope, another step was performed to improve the bond between the TPU envelope and the TPU tubing. A block of aluminum-6061 was machined with a circular channel through its center, with the diameter of the channel equal to the diameter of the tubing. The block was then sawed in half through the channel, and each half was placed on either side of the tubing, sandwiching the tubing between the two sides of the TPU envelope (Figure A.4E). The assembly was then covered temporarily with a PTFE sheet and heat-sealed at 171 °C. Since only the aluminum blocks were in contact with the heating element of the heat press, only the region of the TPU envelope between the blocks was sealed. Thus, a circumferential seal of the TPU envelope onto the tubing was achieved. The jamming envelope was then trimmed to its final form (Figure A.4F).

A.3.2 Repeatability Analysis

Five twenty-layer jamming structures were fabricated. Each sample was placed in a universal materials testing device (Instron 5566, Illinois Tool Works, Norwood, MA) and centered on a static three-point bending fixture (Instron 2810-400) with the supporting anvils (10 mm diameter) set 130 mm apart (Figure A.5A).

Vacuum pressure was controlled using a manual vacuum regulator (EW-07061-30, Cole-Parmer, Vernon Hills, IL). The TPU tubing in each sample was connected to the regulator via highly flexible polyurethane tubing in order to mitigate parasitic loading of the sample by

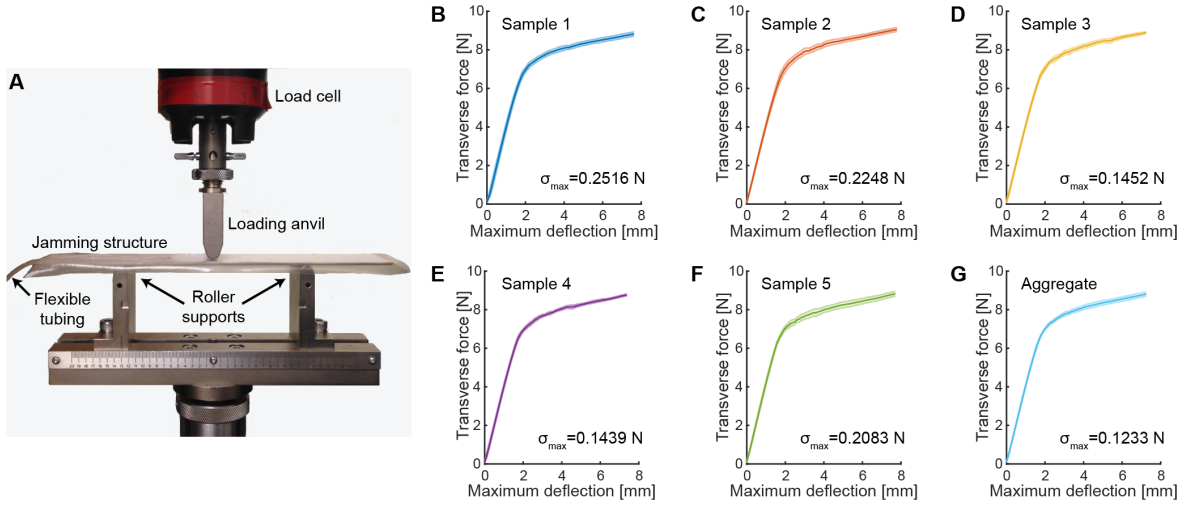


Figure A.5: Testing setup and repeatability analysis for experimental characterization of jamming structures. A) The jamming structure was placed in a test fixture for three-point bending, which consisted of a loading anvil and two roller supports. The loading anvil was attached to a load cell, and the jamming structure was connected to a vacuum regulator via flexible tubing to mitigate parasitic loading of the tubing on the sample. B-F) Testing results for five twenty-layer jamming structures are shown in sequence. For each sample, a mean curve is plotted, along with a shaded error bar that spans ± 1 standard deviation from the mean. The maximum standard deviation at any deflection is given. The structures were highly repeatable from trial to trial. G) The mean curves for all five samples were then aggregated. A mean curve of the mean curves is plotted, along with a shaded error bar. The structures were highly repeatable from sample to sample.

the rigid regulator. Prior to each test, a vacuum pressure of $68 \pm 1.7 \text{ kPa}$ was applied, and a roller was used to remove residual air pockets from the sample.

The loading anvil (10 *mm* diameter) was attached to a 100 *N* load cell (Instron 2525-807) and lowered at a rate of $5 \frac{\text{mm}}{\text{min}}$ until contacting the sample. When the load cell measured a value of 0.010 *N*, the transverse force and displacement of the loading anvil began to be recorded. The anvil was then lowered at a rate of $25 \frac{\text{mm}}{\text{min}}$ until reaching a maximum displacement of 8 *mm*. Tests were conducted at approximately 20% relative humidity. After each test, the sample was disconnected from the regulator and gently flexed multiple times to accelerate its return to ambient pressure. Each sample was tested ten consecutive times.

Occasionally, the loading anvil initially contacted the jamming structure at protruding corners of the seam of its polyurethane envelope; this initial contact caused the materials testing device to undesirably begin measuring force and deflection prior to contacting the bulk of the jamming structure. To discard measurements of the corners of the envelope, we neglected data collected before a small initial force threshold of 0.050 *N* was reached, and we defined zero deflection as the deflection at this threshold. This procedure was always implemented, except for cases in which the force range during a test was comparable to 0.050 *N* (e.g., for five-layer samples in later experimental characterization). No further filtering or smoothing was performed on the raw data.

For each sample, transverse force was plotted against maximum deflection for all ten trials. A mean curve was generated, and standard deviations were computed at each point on the mean curve (Figure A.5B-F). The maximum standard deviation at any deflection was 0.2516 *N*, which constituted 2.881% of the range of the mean curve for that sample. Thus, the mechanical behavior of the structures was highly repeatable from trial to trial, indicating that fatigue was negligible over the examined range of forces and deflections.

The mean curves of all five samples were then aggregated, and a mean curve of the mean curves was generated (Figure A.5G). The maximum standard deviation at any deflection was 0.1233 *N*, which constituted 1.414% of the range of the curve. Thus, the mechanical behavior of the structures was also highly repeatable from sample to sample, demonstrating

that the fabrication process was sufficiently precise. Together, the high trial-to-trial and sample-to-sample repeatability of the jamming structures showed that many samples and trials were not required in order to collect statistically representative data during experimental characterization.

A.3.3 Materials Testing

To provide a fair comparison between experimental and finite element results for many-layer jamming structures, the elastic modulus E and coefficient of friction μ of the copy paper used in the jamming samples were experimentally measured; these values were then used as material properties of the layers in the finite element simulations. The elastic modulus was measured to be approximately 6 *GPa*, and the coefficient of friction was measured to be approximately 0.65. Both properties were measured according to methods outlined in international paper testing standards [100, 101], and the values fell within the ranges reported in literature [74, 102]. The Poisson’s ratio of the copy paper was challenging to measure; thus, a literature value of 0.156 was used instead [74].

A.3.4 Stiffness Characterization Process

The stiffness characterization tests were identical to those conducted for the repeatability analysis (**Experimental Characterization: Repeatability Analysis**). However, fewer samples were tested and fewer trials were executed, as the repeatability analysis showed that many samples and trials were unnecessary. When conducting the tests for the effect of number of layers on stiffness, three samples were fabricated for each number of layers (i.e., three five-layer samples, three ten-layer samples, etc.), and each sample was tested four times at a constant vacuum pressure of 71.1 ± 1.7 *kPa*. When conducting the tests for the effect of vacuum pressure, three twenty-layer samples were made in total, and each sample was tested four times at vacuum pressures of 0, 23.7 ± 1.7 , 47.4 ± 1.7 , and 71.1 ± 1.7 *kPa*. No tests were conducted for the effect of coefficient of friction, as this property could not be precisely varied experimentally.

For each testing group, transverse force was plotted against maximum deflection for all trials. Recall that the effective stiffness k of a jamming structure in three-point bending is equal to the slope of the force-versus-maximum-deflection curve. Again, mean curves were generated, and standard deviations were computed at each point on the mean curve.

A.3.5 Damping Characterization Process

To evaluate finite element predictions for how major design inputs affected the damping of many-layer jamming structures, the damping of jamming structures was experimentally characterized as well. The tests were identical to those conducted for the stiffness characterization. However, after the loading anvil reached its maximum displacement of 8 *mm*, it was then retracted at a rate of 25 $\frac{mm}{min}$ until returning to its original position of 0 *mm*. The transverse force and displacement experienced by the loading anvil continued to be recorded during its retraction. Thus, the behavior of the jamming structures was measured both during loading and unloading.

Due to the extreme similarity of the damping characterization tests to the stiffness characterization tests, a minimal number of samples were tested, and a minimal number of trials were conducted. When conducting the tests for the effect of number of layers on damping, one sample was fabricated for each number of layers, and each sample was tested once at a constant vacuum pressure of $71.1 \pm 1.7 kPa$. When conducting the tests for the effect of vacuum pressure, one twenty-layer sample was made, and the sample was tested once at vacuum pressures of 0, 23.7 ± 1.7 , 47.4 ± 1.7 , and $71.1 \pm 1.7 kPa$.

For each test, transverse force was plotted against maximum deflection for all trials. The unloading of the sample during each test allowed the hysteresis curve to be observed. The area under each hysteresis curve depicted the energy dissipated during the loading cycle, and the effective damping d was simply the energy dissipated per unit deflection. Finite element results for many-layer jamming structures accurately predicted experimental results (Figure A.6). Thus, finite element simulations were not only able to predict the stiffness of many-layer jamming structures, but also their damping behavior.

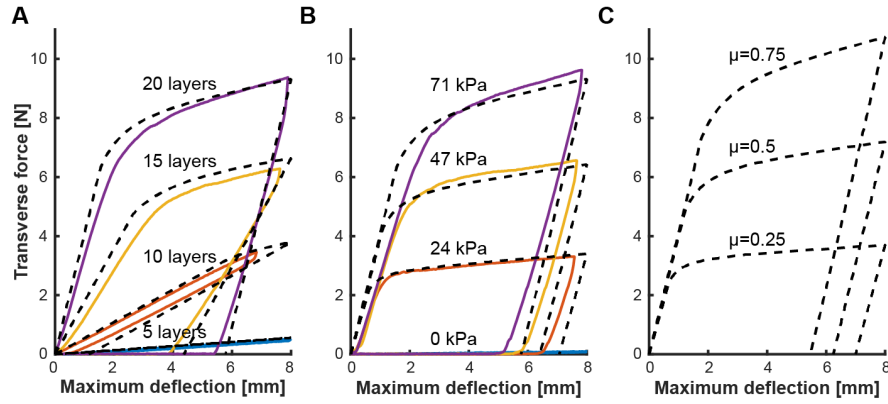


Figure A.6: Finite element predictions and experimental characterization of damping in many-layer jamming structures. Jamming structures were loaded in three-point bending and subsequently unloaded. Transverse force is plotted against maximum deflection; dashed lines indicate finite element predictions, and colored lines denote experimental results. Finite element models accurately predicted experimentally observed hysteresis. The area under the hysteresis curves is equal to the energy dissipated over the loading cycle, and the effective damping d is equal to the energy dissipated per unit deflection. A) The number of layers in the jamming structures was varied. B) Vacuum pressure was varied. No finite element data is provided for the 0 kPa case, as the model was unstable. C) Coefficient of friction was varied. No experimental data is shown, as coefficients of friction could not be precisely varied experimentally.

A.4 Functions and Applications

All molds for the subsequent demonstrations were designed using CAD software (SolidWorks 2015, Dassault Systèmes, Villacoublay, France) and 3D printed using a stereolithography-based printer (Objet30 Scholar, Stratasys, Ltd., Eden Prairie, MN).

A.4.1 Shape-Locking

A soft pneumatic bending actuator was designed and fabricated based on previous literature [14, 103]. The top of the actuator (i.e., the inflatable chambers) was cast using a two-part mold, whereas the bottom (i.e., a thick, flat layer to promote bending rather than extension) was cast using a one-part mold. All parts were cast from shore 10A platinum-cure silicone rubber (Dragon Skin 10 Medium, Smooth-On, Inc., Macungie, PA). A twenty-layer jamming structure was then designed and fabricated using the techniques described earlier (**Experimental Characterization: Fabrication Process**). The structure spanned the ventral surface of the actuator (i.e., the longitudinal surface with the smaller radius of curvature when the actuator

was pressurized). Finally, the actuator and jamming structure were bonded using silicone building sealant (Dow Corning 795, Dow Corning, Midland, MI).

The actuator and jamming structure were connected to pressure and vacuum inputs, respectively. The pressure source was regulated by a digital pressure regulator (ITV1031, SMC Pneumatics, Yorba Linda, CA), whereas the vacuum source was regulated by the device used in experimental characterization. The output from each of the regulators passed through two miniature pneumatic solenoid valves (V^2 Valves, Parker Hannifin, Hollis, NH) before entering the actuator and jamming structure. The valves were controlled by pushbuttons and enabled the actuator and jamming structure to each have three states: a pressurizing (or vacuuming) state, a hold state where the internal pressure (or vacuum) is preserved, and a depressurizing (or vacuum-relieving) state.

The actuator was pressurized to 16 kPa , and a photograph was taken perpendicular to the bending plane. Two tests were then conducted. In the first test, the actuator was depressurized to 0 kPa . In the second test, a vacuum of 85 kPa was first applied to the jamming structure, and the actuator was then depressurized to 0 kPa . A photograph was again taken once the system came to rest.

For each photograph, the arc of the ventral surface of the shape-locking system was digitally traced. The data points comprising each arc were then interpolated over 100 equally spaced points. The coefficient of determination (R^2) value was computed between the two interpolated arcs.

A.4.2 Variable Kinematics

The substrate of the variable kinematics system was fabricated according to the same process used for the actuator component of the shape-locking system. However, the substrate was cast using a one-part mold with an inserted hexagonal rod, which created a channel to route a cable; furthermore, the substrate was cast from high-stiffness PDMS rubber (Sylgard 184, Dow Corning, Midland, MI). The jamming structure was also designed and fabricated according to the techniques described earlier, but with three distinct stacks of twenty strips separated

by 1 *mm* gaps within the TPU envelope. The jamming structure and rubber substrate were again bonded with silicone building sealant (Dow Corning 795, Dow Corning, Midland, MI).

Braided polyethylene cable (Hollow Spectra, BHP Tackle, Harrington Park, NJ) was then routed through the channel in the substrate. The cable was tied at one end to a turnbuckle and at the other end to a small washer. During testing, the turnbuckles were manually twisted, which pulled the cable, compressed the washer against the end of the variable kinematics structure, and induced bending.

A.4.3 Two-Fingered Grasper

Each fingertip in the two-fingered grasper had a cylindrical surface with a radius of 5 *mm*. The fingertips were cast using a two-part mold according to the same process used for the shape-locking actuator and variable-kinematics substrate; however, the fingertips were cast from shore 00-10A silicone rubber (Ecoflex 00-10, Smooth-On, Inc., Macungie, PA).

To test bending stiffness and off-axis bending stiffness, the cable was removed from the finger being tested, and the finger was clamped in the vertical position. A digital force gauge (Chatillon DFI10, AMETEK Sensors, Test & Calibration, Berwyn, PA) was moved along a rigid guiderail until contacting the finger 25 *mm* from its distal end. The force gauge was then pushed forward in 12.5 *mm* increments. To mitigate viscous effects, approximately five seconds were allowed to elapse, and the force measurement was then recorded. The test was conducted with vacuum off and vacuum on. When measuring bending stiffness, the force gauge was pushed in the direction of the thickness of the substrate; when measuring off-axis bending stiffness, the gauge was in the direction of the width. Because the minimum force measurable by the force gauge was 0.05 *N*, any reading of 0.00 *N* was rounded to 0.05 *N*; thus, the stiffness increases reported in the main text were worst-case (i.e., lowest possible) estimates.

To measure the torsional stiffness of the fingers in the two-fingered grasper, a custom testing device was designed and fabricated. In the device, a finger was coupled to a pulley with a radius of 16.25 *mm*, which itself was coupled via a cable to a digital force gauge (Chatillon

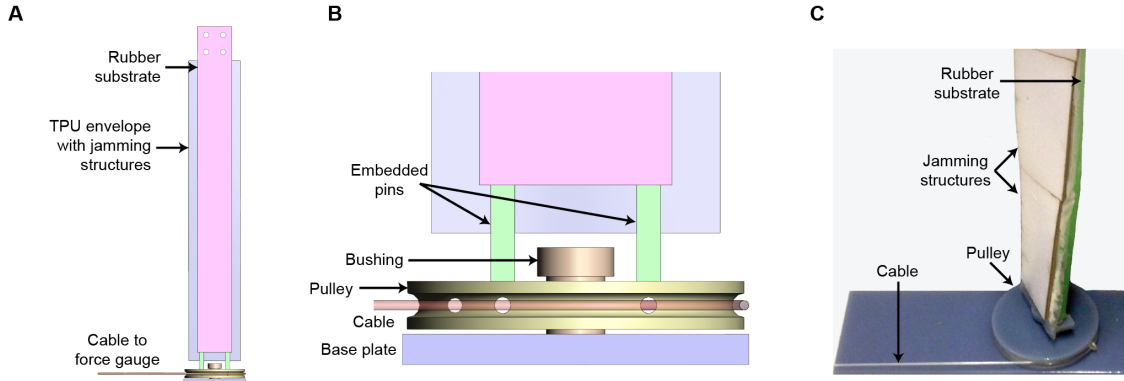


Figure A.7: Testing device for measuring the torsional stiffness of a variable kinematics system. *A) Front view of the testing device. A variable kinematics structure is hung vertically. Its upper end is bolted in place, and its lower end is coupled to a cable that is connected to a force gauge. B) Zoom view of the lower end of the testing device. Two plastic pins are embedded in the variable kinematics structure and fastened to a pulley, and the cable is wound around the pulley. When the cable is pulled, the pulley spins around a bushing, and the variable kinematics structure is twisted about its longitudinal axis. C) Physical implementation of the testing device. The variable kinematics structure was originally oriented as shown in A and B; its lower end has now been twisted by approximately 80° .*

DFI10 AMETEK Sensors, Test & Calibration, Berwyn, PA) (Figure A.7). Aside from the finger, cable, and force gauge, all components of the device were 3D printed (Objet30 Scholar, Stratasys, Ltd., Eden Prairie, MN). When the force gauge was pulled, the finger was twisted about its longitudinal axis.

The force gauge was retracted in 12.5 mm increments, and force measurements were recorded at each increment. The force measurements were multiplied by the radius of the pulley to calculate torque. Torque was then plotted against pull distance, and the slope was calculated to quantify the torsional stiffness of the finger.

A.5 Additional Concepts

A.5.1 Continuously-Variable Stiffness

As first outlined in a previous study [34], continuously-variable stiffness can be achieved by stacking multiple jamming structures that have independent vacuum inputs (Figure A.8A). The bending stiffness of the composite structure is determined by the number of jamming structures that have vacuum applied (Figure A.8B). If the layers are compliant and the number of layers

within each jamming structure is small, the bending stiffness of the composite structure can be selected with high resolution.

Many schemes are possible for distributing layers across the jamming structures. One particularly appealing scheme is a binary distribution (i.e., one 2-layer structure, one 4-layer structure, one 8-layer structure, and so on). With such a scheme, a high dynamic range (i.e., the ratio of the stiffness range to the stiffness resolution) can be achieved.

To demonstrate this behavior, a hypothetical case study was conducted (Figure A.8C). Consider a continuously-variable stiffness structure consisting of thirty layers. Let k be the bending stiffness of a single layer. Consider the following three methods for distributing the layers across jamming structures: 1) the layers are distributed by binary numbering across four jamming structures (i.e., one two-layer jamming structure, one four-layer, one eight-layer, and one sixteen-layer), 2) the layers are distributed nearly equitably across four jamming structures (i.e., two seven-layer structures and two eight-layer structures), and 3) the layers are distributed equitably across fifteen jamming structures (i.e., fifteen two-layer structures).

The four-structure binary scheme has the superior stiffness range, best-case resolution, and maximum dynamic range, as well as a high number of unique stiffness values; furthermore, with four vacuum inputs, it is simple to physically implement. The fifteen-structure equitable scheme has superior worst-case resolution and minimum dynamic range, as well as the highest number of unique stiffness values; on the other hand, with fifteen inputs, it is challenging to implement. For most applications, a binary scheme may be preferred.

A.5.2 Continuously-Variable Damping

From finite element analysis, it was found that the full-slip damping of a many-layer jamming structure scales linearly with vacuum pressure. Thus, continuously-variable damping can be achieved simply by varying the vacuum pressure on a single jamming structure (Figure A.8D).

Finite element analysis also showed that the full-slip damping of a many-layer jamming structure scales linearly with the number of layers. In practical applications where high damping is desired but the pressure gradient is limited (e.g., with vacuum, where the gradient

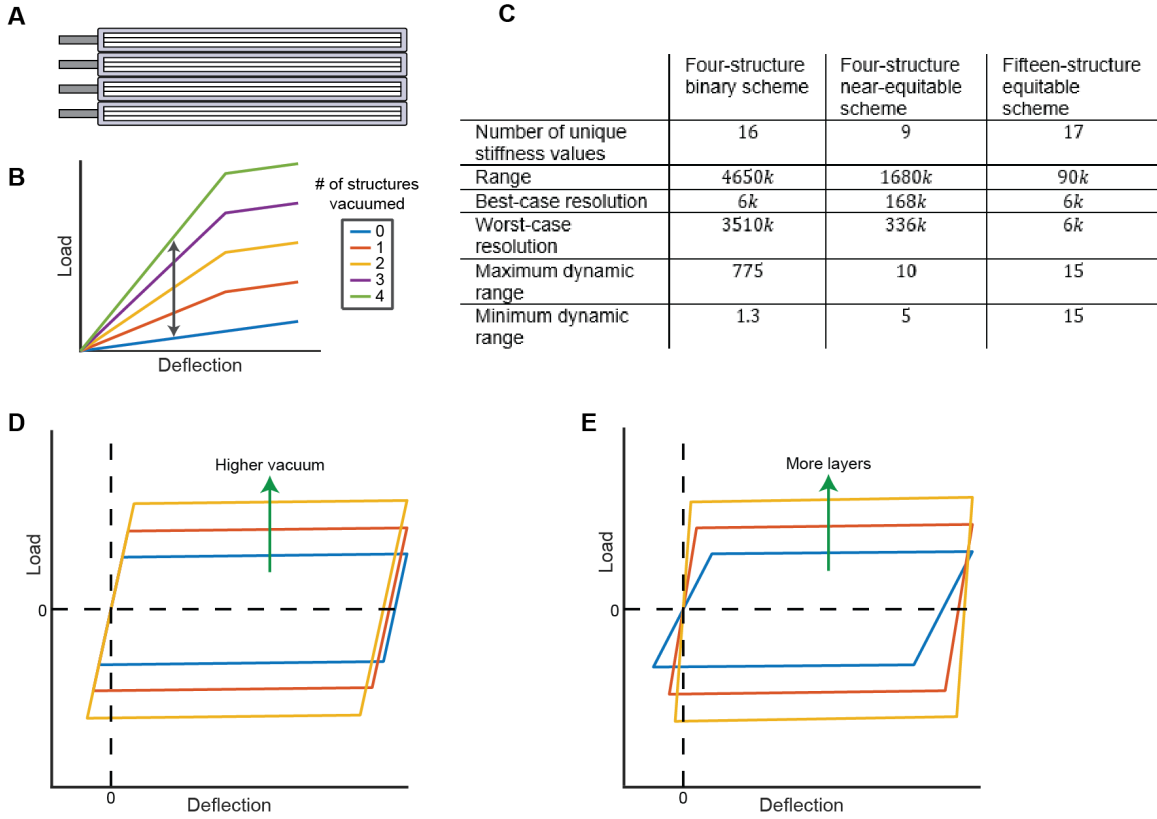


Figure A.8: Conceptual examples of continuously-variable stiffness and damping structures. For simplicity, transition regimes between the pre-slip regime and the full-slip regime are not depicted. A) In one implementation of continuously-variable stiffness, four jamming structures are stacked and bonded. Each jamming structure has an independent vacuum input and contains three layers of compliant material. B) By applying or relieving vacuum from individual jamming structures, the pre-slip bending stiffness of the composite structure may be selected from one of five possible values. C) For a continuously-variable stiffness structure consisting of thirty total layers, three different ways are considered for distributing the layers across multiple jamming structures. Quantity k is the bending stiffness of a single layer. A four-structure binary scheme is preferable over equitable and near-equitable schemes, as it has the best stiffness range, resolution, and dynamic range, and it is simple to physically implement. D) Conceptual load-versus-deflection curves are shown for a continuously-variable damping structure. (Because it may be desirable to use such a structure over multiple cycles, a full hysteresis loop is shown; the structure is loaded, unloaded, and then loaded and unloaded in the opposite direction to return to zero deflection.) Increasing the vacuum pressure augments the dissipated energy (i.e., the area enclosed by the hysteresis loop) and damping (i.e., the dissipated energy per unit deflection). E) Increasing the number of layers again augments damping. Furthermore, it minimizes the range of deformation over which the pre-slip regime occurs, maximizing the range over which damping is nonzero.

is limited to the ambient pressure), the maximum damping value of a continuously-variable damping structure can be augmented in advance by increasing the number of layers in the jamming structure during fabrication (Figure A.8E). Note that the maximum damping value can also be augmented by increasing the coefficient of friction of the layers.

In other practical applications, damping may be desired over the full range of deformation of the jamming structure. However, as described earlier, damping of a many-layer jamming structure is zero during pre-slip, creating a dead zone for damping; thus, the deformation range over which pre-slip occurs should be minimized. As also described earlier, the pre-slip stiffness of a many-layer jamming structure scales with n^3 , whereas the load at which the structure begins to slip (i.e., the maximum load of pre-slip) scales with n (Finite Element Modeling: Functional Dependencies). In turn, the deformation at which the structure begins to slip (i.e., the maximum deformation of pre-slip) scales with n^{-2} . Thus, the deformation range over which pre-slip occurs can be minimized in advance by again increasing the number of layers in the jamming structure during fabrication (Figure A.8E).

In conclusion, continuously-variable damping can be achieved by simply varying the vacuum pressure on a many-layer jamming structure. In practical applications that require high energy dissipation over a maximal range of displacements with a minimal dead zone, such a structure should be fabricated with as many layers as possible.

A.5.3 Spring-Based Jamming

As an alternative to fluidic and electrostatic means for actuating jamming structures, a spring-based actuation method can be implemented (Figure A.9). In this method, elastic elements (e.g., spring clips) are arranged along the length of a jamming structure that is enclosed in an airtight envelope. In its default state, the elastic elements cause the layers in the jamming structure to be cohesive, and the structure is stiff. However, when the airtight envelope is pressurized, the elastic elements are pushed apart, allowing the layers to slip freely; the jamming structure is compliant and can be reconfigured.

A spring-based actuation method has two distinct advantages. First, the maximum

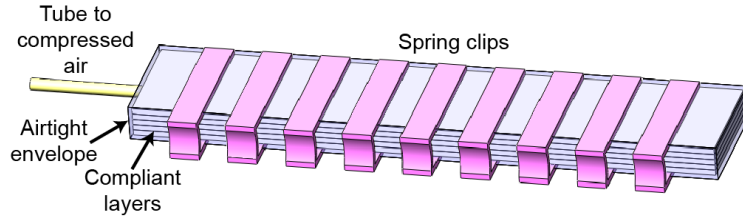


Figure A.9: *Conceptual example of a spring-based jamming structure. A many-layer jamming structure enclosed in an airtight envelope is connected to a compressed air source and has spring clips arranged along its length. When no air is supplied, the spring clips cause the layers to be cohesive, and the structure is stiff. When air is supplied, the clips are pushed open, and the layers can slip freely. The structure is then compliant and may be reconfigured.*

frictional stress at the interfaces between layers can be set to arbitrarily high values by using clips with a higher (or adjustable) spring constant. As described earlier, the load at which a jamming structure begins to slip scales with the pressure gradient P ([Finite Element Modeling: Functional Dependencies](#)); thus, the structure can maintain its pre-slip stiffness over larger loads than a jamming structure that is actuated by vacuum pressure. Second, the structure only requires power to change its shape, not to preserve it. In applications where the time spent reshaping the jamming structure is much smaller than the time spent locked in a particular configuration, this actuation mechanism can expend negligible energy.

Appendix B

Appendix to Chapter 4

B.1 Experimental Proof-of-Concept

B.1.1 Materials and Material Processing

- **Steel:** Stock consisted of 30 *cm* x 20 *cm* x 0.05 *mm* (12" x 8" x 0.002") low-carbon steel flat shim stock (9011K221, McMaster-Carr, Elmhurst, IL). Sheets were cut using a metal shear.
- **Paper:** Stock consisted of 28 *cm* x 43 *cm* x 0.1 *mm* (11" x 17" x 0.004") sheets of copy paper (Ultra White Multipurpose Copy Paper, HP Inc., Palo Alto, CA). Sheets were cut using a laser-cutter (VLS4.60, Universal Laser Systems, Inc., Scottsdale, AZ).
- **Low-density polyethylene (LDPE):** Stock consisted of a 15 *m* x 90 *cm* x 0.1 *mm* (50 *ft* x 3 *ft* x 0.004") roll (8593K71, McMaster-Carr, Elmhurst, IL). Roll was manually cut using razor blades, as laser-cutting produced rough edges.
- **Polyurethane (PU) foam:** Stock consisted of 30 *cm* x 30 *cm* x 0.8 *mm* (12" x 12" x $\frac{1}{32}$ ") sheets (86375K131, McMaster-Carr, Elmhurst, IL). Sheets were cut using a laser-cutter (VLS4.60, Universal Laser Systems).

B.1.2 Sample Fabrication

The following samples were constructed:

- **Steel-paper:** 2 layers of steel, {20, 25, 30, 35} layers of paper, and 2 layers of steel, with dimensions of 30 *cm* x 2.5 *cm* (12" x 2"). Samples were enclosed in a tear-resistant TPE envelope (Stretchlon 800, Fibre Glast Developments Corp., Brookville, OH) with a thickness of 0.05 *mm* (2 *mil*).
- **Steel-LDPE:** 2 layers of steel, 20 layers of LDPE, and 2 layers of steel, with dimensions of 30 *cm* x 2.5 *cm* (12" x 2"). Samples were enclosed in a tear-resistant TPE envelope (Stretchlon 800, Fibre Glast Developments Corp.) with a thickness of 0.05 *mm* (2 *mil*). Note that due to the difficulty of manually cutting the LDPE layers, only 1 number of core layers was tested.
- **Paper-foam:** 2 layers of paper, {5, 6, 7, 8} layers of PU foam, and 2 layers of paper, with dimensions of 15 *cm* x 2.5 *cm* (5.75" x 2"). Samples were enclosed in a flexible TPE envelope (Stretchlon 200, Fibre Glast Developments Corp.) with a thickness of 0.04 *mm* (1.5 *mil*).

Two samples of each layer configuration were constructed. For each sample, the layers were stacked and enclosed within the corresponding TPE envelope. All four edges of the envelope were sealed using an impulse sealer (AIE-450FD, American International Electric Inc., City of Industry, CA). A small hole was cut in the envelope, and a thermoplastic polyurethane (TPU) tube with outer diameter of 3 *mm* (0.125") was inserted (TPU1-2N, Eldon-James Corp., Denver, CO). Polytetrafluoroethylene (PTFE) thread seal tape was then wrapped around the interface between the TPU tube and the TPE envelope to ensure an airtight seal. Photographs and videos of an equivalent fabrication process for standard laminar jamming structures have been published to the Soft Robotics Toolkit, an open-access educational website [104].

B.1.3 Sample Testing

All samples were tested on a 3-point bending fixture (Instron 2810-400, Illinois Tool Works, Norwood, MA) in a universal materials testing device (Instron 5566, Illinois Tool Works) with a loading anvil attached to a load cell with a 100 N force capacity (Instron 2525-807, Illinois Tool Works).

Prior to each test, the supporting anvils were separated by 15 cm , 15 cm , and 9 cm for the steel-paper, steel-LDPE, and paper-foam samples, respectively, in order to prevent them from bulging or sagging in the unjammed state. The samples were connected to a vacuum source that was regulated to 71 kPa (21 $inHg$) using a manual vacuum regulator (EW-07061-30, Cole-Parmer, Vernon Hills, IL). The samples were then flattened using a rolling pin in order to remove large air pockets.

To ensure that the deflection of the samples was measured with respect to initial contact, the samples were preloaded at 5 $\frac{mm}{min}$ to a nominal force value of 0.1 N ; at this point, deflection measurements were zeroed. The samples were then loaded at 20 $\frac{mm}{min}$ to a maximum deflection of 5 mm , 2 mm , and 5 mm for the steel-paper, steel-LDPE, and paper-foam samples, respectively. They were unloaded at the same rate. Force and deflection values were recorded during loading and unloading. After each test, the samples were released from vacuum and gently flexed to facilitate their return to atmospheric pressure. Tests were repeated 10 times per sample.

B.1.4 Data Processing

Raw data were imported into mathematical analysis software (MATLAB 2018a, MathWorks, Natick, MA). For each layer configuration, data from both samples and all trials were averaged to produce a mean force-versus-deflection curve. Error bars corresponding to ± 1 standard deviation were illustrated at each point along the curve using the *shadedErrorBar* package [105]. Cubic splines with 20 knots were fit to the loading portion of the curve. Yield forces were determined by manually estimating the end of the initial linear regime. Stiffnesses were determined by computing first derivatives of the spline fits in the initial linear regime.

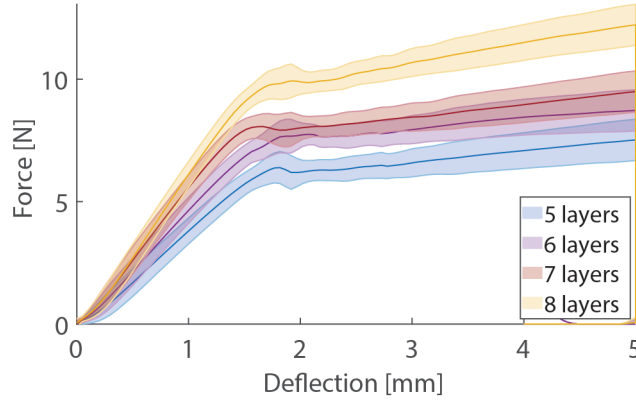


Figure B.1: Force-versus-deflection curves for paper-foam sandwich jamming structures in 3-point bending at 71 kPa vacuum pressure. Curves are shown for different numbers of core layers. Each curve is a mean curve from 2 samples and 10 trials; shaded error bars delimit ± 1 standard deviation.

Table B.1: Performance-to-mass improvements for three material configurations examined during experimental proof-of-concept of sandwich jamming structures. Recall from *Sample Fabrication*, 4 different numbers of core layers were tested, whereas for the steel-LDPE configuration, 1 number of core layers (i.e., 20) was tested. For each maximum improvement ratio, the corresponding number of layers is listed. Note that steel-paper sandwich jamming structures had by far the best stiffness-to-mass and yield-to-mass improvement ratios, as well as the second-best range-to-mass improvement ratio.

Material Configuration	Maximum Stiffness-to-Mass Improvement	# of Layers	Maximum Range-to-Mass Improvement	# of Layers	Maximum Yield-to-Mass Improvement	# of Layers
Steel-Paper	176	35	65	30	15	35
Steel-LDPE	18	20	132	20	0.7	20
Paper-Foam	7	7	5	5	0.7	8

B.1.5 Additional Data

Figure B.1 shows experimental force-versus-deflection data for the paper-foam sandwich jamming structures. Furthermore, Table B.1 shows a comparison of improvement ratios extracted from force-versus-deflection curves for all examined material configurations.

B.2 Analytical Modeling

B.2.1 Derivation of Improvement Ratios

Stiffness, Range, and Yield of a Single-Material Laminar Jamming Structure

For a single-material laminar jamming structure with a rectangular cross-section in the jammed state, the bending stiffness is

$$k_b = EI = \frac{EbH^3}{12} \quad (\text{B.1})$$

where E is the elastic modulus, I is the second moment of area (also referred to as the “area moment of inertia”), b is the width of the structure, and H is the total thickness of the structure [49].

The range is

$$r = n^2 \quad (\text{B.2})$$

where n is the number of layers [49].

The yield force is

$$F_{crit} = \frac{4bH\mu P}{3} \quad (\text{B.3})$$

where μ is the coefficient of friction of the layers and P is the vacuum pressure (i.e., the absolute pressure applied to the structure below ambient pressure) [49].

Stiffness, Range, and Yield of a Sandwich Jamming Structure

For a cohesive sandwich structure, the bending stiffness is

$$k_b = \frac{E_c c^3}{12} + \frac{E_f (\frac{f}{2})(c + \frac{f}{2})^2}{2} + \frac{E_f (\frac{f}{2})^3}{6} \quad (\text{B.4})$$

where E_c and E_f are the elastic moduli of the core and face, respectively; and c and f are the total thickness of the core and both faces, respectively [83]. If the faces are thin (i.e., $f \ll c$)

and the core is compliant (i.e., $E_c \ll E_f$), then the bending stiffness can be approximated as

$$k_b \approx E_f b \frac{\frac{f}{2}(c + \frac{f}{2})^2}{2} = E_f b \frac{4c^2 f + 4cf^2 + f^3}{16} \quad (\text{B.5})$$

To calculate the range, we first compute the jammed bending stiffness. This stiffness is simply equal to that of a cohesive sandwich structure given in (B.5). Assume that the sandwich jamming structure consists of n_c core layers, each of height h_c , and n_f total face layers, each of height h_f . Thus, $c = n_c h_c$ and $f = n_f h_f$. Substituting these expressions into (B.5), the jammed bending stiffness can be written as

$$k_b^{jam} \approx E_f b \frac{4(n_c h_c)^2 f + 4(n_c h_c)(n_f h_f)^2 + (n_f h_f)^3}{16} \quad (\text{B.6})$$

Next, we calculate the unjammed bending stiffness. This stiffness is simply equal to the sum of the individual stiffnesses of all the layers. Thus, the unjammed bending stiffness is

$$k_b^{unjam} = b \frac{E_c n_c h_c^3 + E_f n_f h_f^3}{12} \quad (\text{B.7})$$

The range is the ratio of the jammed to unjammed bending stiffnesses. Thus, the range is

$$r = E_f \frac{12(n_c h_c)^2 (n_f h_f) + 12(n_c h_c)(n_f h_f)^2 + 3(n_f h_f)^3}{4(E_c n_c h_c^3 + E_f n_f h_f^3)} \quad (\text{B.8})$$

The yield force is calculated by equating the maximum longitudinal shear stress with the maximum allowable shear stress. Again, for thin faces and a compliant core, the shear stress is approximately negligible in the faces and uniformly distributed in the core. The maximum longitudinal shear stress is thus

$$\tau_{max} = \frac{V}{A} = \frac{V}{bc} \quad (\text{B.9})$$

where V is the resultant shear and A is the cross-sectional area. In 3-point bending, $V = \frac{F}{2}$, where F is the applied load. Thus,

$$\tau_{max} = \frac{F}{2bc} \quad (\text{B.10})$$

The maximum allowable shear stress is simply

$$\tau_{allow} = \mu_c P \quad (\text{B.11})$$

where μ_c is the coefficient of friction of the core material. Equating (B.10) and (B.11) and solving for F , the yield force is

$$F_{crit} = 2bc\mu_c P \quad (\text{B.12})$$

Note that if $\mu_{cf} < \mu_c$, where μ_{cf} is the coefficient of friction between the core and the face, then μ_{cf} should be used in the preceding expression, as the structure will begin to slip at the interfaces between the core and face.

Equal-Material Improvement Ratios

Bending Stiffness: The bending stiffness of a sandwich jamming structure is

$$k_b \approx E_f b \frac{4c^2 f + 4cf^2 + f^3}{16} \quad (\text{B.13})$$

The mass is

$$m = \rho_c bcL + \rho_f b f L \quad (\text{B.14})$$

where m is the mass, ρ_c is the density of the core material, ρ_f is the density of the face material, and L is the length. Thus, the stiffness-to-mass ratio of a sandwich jamming structure is

$$\frac{k_b}{m} \approx E_f \frac{4c^2 f + 4cf^2 + f^3}{16L(\rho_c c + \rho_f f)} \quad (\text{B.15})$$

A single-material jamming structure composed of just the face material (i.e., an equal-material jamming structure) has thickness f . Thus, the bending stiffness of an equal-material jamming structure is

$$k_b = E_f \frac{bf^3}{12} \quad (\text{B.16})$$

The mass is

$$m = \rho_f b f L \quad (\text{B.17})$$

Thus, the stiffness-to-mass ratio is

$$\frac{k_b}{m} = E_f \frac{f^2}{12L\rho_f} \quad (\text{B.18})$$

Thus, the ratio of the stiffness-to-mass-ratios of a sandwich jamming structure and that of an equal-material jamming structure is

$$\left(\frac{k_b}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (\text{B.19})$$

Range: The range of a sandwich jamming structure is

$$r = E_f \frac{12(n_ch_c)^2(n_fh_f) + 12(n_ch_c)(n_fh_f)^2 + 3(n_fh_f)^3}{4(E_cn_ch_c^3 + E_fn_fh_f^3)} \quad (\text{B.20})$$

The mass is

$$m = \rho_c bcL + \rho_f b f L \quad (\text{B.21})$$

Thus, the range-to-mass ratio is

$$\frac{r}{m} = E_f \frac{12(n_ch_c)^2(n_fh_f) + 12(n_ch_c)(n_fh_f)^2 + 3(n_fh_f)^3}{4bL(E_cn_ch_c^3 + E_fn_fh_f^3)(\rho_cc + \rho_ff)} \quad (\text{B.22})$$

The range of an equal-material jamming structure is

$$r = n_f^2 \quad (\text{B.23})$$

where n_f is the number of layers in the face of the sandwich jamming structure.

The mass is

$$m = \rho_f b f L \quad (\text{B.24})$$

Thus, the range-to-mass ratio is

$$\frac{r}{m} = \frac{n_f^2}{bL\rho_ff} = \frac{n_f}{bL\rho_fh_f} \quad (\text{B.25})$$

Thus, the ratio of the range-to-mass-ratios of a sandwich jamming structure and that of an equal-material jamming structure can be written as

$$\left(\frac{r}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\frac{E_c}{E_f}\left(\frac{h_c}{h_f}\right)^2\frac{c}{f} + 1\right)\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (\text{B.26})$$

Yield: The yield of a sandwich jamming structure is

$$F_{crit} = 2bc\mu_c P \quad (\text{B.27})$$

The mass is

$$m = \rho_c bcL + \rho_f bfL \quad (\text{B.28})$$

Thus, the yield-to-mass ratio is

$$\frac{F_{crit}}{m} = \frac{2\mu_c P}{L(\rho_c + \rho_f(\frac{c}{f})^{-1})} \quad (\text{B.29})$$

The yield of an equal-material jamming structure is

$$F_{crit} = \frac{4bf\mu_f P}{3} \quad (\text{B.30})$$

The mass is

$$m = \rho_f bfL \quad (\text{B.31})$$

Thus, the yield-to-mass ratio is

$$\frac{F_{crit}}{m} = \frac{4\mu_f P}{3L\rho_f} \quad (\text{B.32})$$

Thus, the ratio of the yield-to-mass-ratios of a sandwich jamming structure and that of an equal-material jamming structure is

$$\left(\frac{F_{crit}}{m} \right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1} \quad (\text{B.33})$$

Equal-Mass Improvement Ratios

Because the masses are the same, the ratio of stiffness-to-mass ratios is simply equal to the ratio of stiffness, and so on for the other performance metrics.

Bending Stiffness: The bending stiffness of a sandwich jamming structure is

$$k_b \approx E_f b \frac{4c^2 f + 4cf^2 + f^3}{16} \quad (\text{B.34})$$

The mass of the sandwich jamming structure is the same as that of the equal-mass jamming structure. Thus,

$$\rho_c b c L + \rho_f b f L = \rho_f b H L \quad (\text{B.35})$$

$$H = \frac{\rho_c}{\rho_f} c + f \quad (\text{B.36})$$

where H is the total thickness of the structure. Thus, the stiffness of an equal-mass jamming structure is

$$k_b = E_f \frac{b(\frac{\rho_c}{\rho_f} c + f)^3}{12} = E_f b \frac{(\frac{\rho_c}{\rho_f})^3 c^3 + 3(\frac{\rho_c}{\rho_f})^2 c^2 f + 3\frac{\rho_c}{\rho_f} c f^2 + f^3}{12} \quad (\text{B.37})$$

Thus, the ratio of the stiffness-to-mass-ratios of a sandwich jamming structure and that of an equal-mass jamming structure is

$$\left(\frac{k_b}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4(\frac{\rho_c}{\rho_f})^3(\frac{c}{f})^3 + 12(\frac{\rho_c}{\rho_f})^2(\frac{c}{f})^2 + 12\frac{\rho_c}{\rho_f}\frac{c}{f} + 4} \quad (\text{B.38})$$

Range: The range of a sandwich jamming structure is

$$r = E_f \frac{12(n_c h_c)^2 (n_f h_f) + 12(n_c h_c)(n_f h_f)^2 + 3(n_f h_f)^3}{4(E_c n_c h_c^3 + E_f n_f h_f^3)} \quad (\text{B.39})$$

Again, the thickness of an equal-mass jamming structure is

$$H = \frac{\rho_c}{\rho_f} c + f \quad (\text{B.40})$$

Thus, the range of an equal-mass jamming structure is

$$r = n_f^2 = \left(\frac{H}{h_f}\right)^2 = \frac{(\frac{\rho_c}{\rho_f})^2 (n_c h_c)^2 + 2\frac{\rho_c}{\rho_f} (n_c h_c)(n_f h_f) + (n_f h_f)^2}{h_f^2} \quad (\text{B.41})$$

where it is assumed that h_f for the equal-mass jamming structure is the same as that for the sandwich jamming structure.

Thus, the ratio of the range-to-mass-ratios of a sandwich jamming structure and that of an

equal-mass jamming structure is

$$\left(\frac{r}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\frac{E_c}{E_f}\left(\frac{h_c}{h_f}\right)^2\frac{c}{f} + 1\right)\left(\left(\frac{\rho_c}{\rho_f}\right)^2\left(\frac{c}{f}\right)^2 + 2\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (\text{B.42})$$

Yield: The yield of a sandwich jamming structure is

$$F_{crit} = 2bc\mu_c P \quad (\text{B.43})$$

Again, the thickness of an equal-mass jamming structure is

$$H = \frac{\rho_c}{\rho_f}c + f \quad (\text{B.44})$$

Thus, the yield of an equal-mass jamming structure is

$$F_{crit} = \frac{4b\left(\frac{\rho_c}{\rho_f}c + f\right)\mu_f P}{3} \quad (\text{B.45})$$

Thus, the ratio of the yield-to-mass-ratios of a sandwich jamming structure and that of an equal-mass jamming structure is

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f}\frac{c}{f} + 1} \quad (\text{B.46})$$

Equal-Volume Improvement Ratios

Bending Stiffness: The stiffness-to-mass ratio of a sandwich jamming structure is

$$\frac{k_b}{m} \approx E_f \frac{4c^2f + 4cf^2 + f^3}{16L(\rho_c c + \rho_f f)} \quad (\text{B.47})$$

The volume of the sandwich jamming structure is the same as that of the equal-volume jamming structure. Thus,

$$H = c + f \quad (\text{B.48})$$

Thus, the stiffness-to-mass ratio of an equal-volume jamming structure is

$$\frac{k_b}{m} = E_f \frac{(c + f)^2}{12L\rho_f} = E_f \frac{c^2 + 2cf + f^2}{12L\rho_f} \quad (\text{B.49})$$

Thus, the ratio of the stiffness-to-mass-ratios of a sandwich jamming structure and that of an equal-volume jamming structure is

$$\left(\frac{k_b}{m}\right)^* = \frac{12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3}{4\left(\left(\frac{c}{f}\right)^2 + 2\frac{c}{f} + 1\right)\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (\text{B.50})$$

Range: The range-to-mass ratio of a sandwich jamming structure is

$$\frac{r}{m} = E_f \frac{12(n_ch_c)^2(n_fh_f) + 12(n_ch_c)(n_fh_f)^2 + 3(n_fh_f)^3}{4bL(E_cn_ch_c^3 + E_fn_fh_f^3)(\rho_cc + \rho_ff)} \quad (\text{B.51})$$

Again,

$$H = c + f \quad (\text{B.52})$$

Thus, the range-to-mass ratio of an equal-volume jamming structure is

$$\frac{r}{m} = \frac{n_f^2}{bL\rho_f(c + f)} \quad (\text{B.53})$$

Thus, the ratio of the range-to-mass-ratios of a sandwich jamming structure and that of an equal-volume jamming structure is

$$\frac{(12\left(\frac{c}{f}\right)^2 + 12\frac{c}{f} + 3)\left(\frac{c}{f} + 1\right)}{4\left(\frac{E_c}{E_f}\left(\frac{h_c}{h_f}\right)^2\frac{c}{f} + 1\right)\left(\frac{\rho_c}{\rho_f}\frac{c}{f} + 1\right)} \quad (\text{B.54})$$

Yield: The yield-to-mass ratio of a sandwich jamming structure is

$$\frac{F_{crit}}{m} = \frac{2\mu_c P}{L(\rho_c + \rho_f\left(\frac{c}{f}\right)^{-1})} \quad (\text{B.55})$$

The yield-to-mass ratio of an equal-volume jamming structure is

$$\frac{F_{crit}}{m} = \frac{4\mu_f P}{3L\rho_f} \quad (\text{B.56})$$

Note that this expression does not depend on the height of the structure, as both the yield and the mass are proportional to the height. Thus, the ratio of the yield-to-mass-ratios of a

sandwich jamming structure and that of an equal-volume jamming structure is

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1} \quad (\text{B.57})$$

which is the same as the previous expressions.

Summary of Formulae

The final results from above are collected and rewritten below in comparable forms.

For equal material,

$$\left(\frac{k_b}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1)} \quad (\text{B.58})$$

$$\left(\frac{r}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4(\frac{E_c}{E_f}(\frac{h_c}{h_f})^2 \frac{c}{f} + 1)(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1)} \quad (\text{B.59})$$

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1} \quad (\text{B.60})$$

For equal mass,

$$\left(\frac{k_b}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1)^3} \quad (\text{B.61})$$

$$\left(\frac{r}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4(\frac{E_c}{E_f}(\frac{h_c}{h_f})^2 \frac{c}{f} + 1)(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1)^2} \quad (\text{B.62})$$

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1} \quad (\text{B.63})$$

For equal volume,

$$\left(\frac{k_b}{m}\right)^* = \frac{12(\frac{c}{f})^2 + 12\frac{c}{f} + 3}{4\left((\frac{c}{f})^2 + 2\frac{c}{f} + 1\right)\left(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1\right)} \quad (\text{B.64})$$

$$\left(\frac{r}{m}\right)^* = \frac{(12(\frac{c}{f})^2 + 12\frac{c}{f} + 3)(\frac{c}{f} + 1)}{4(\frac{E_c}{E_f}(\frac{h_c}{h_f})^2 \frac{c}{f} + 1)(\frac{\rho_c}{\rho_f} \frac{c}{f} + 1)} \quad (\text{B.65})$$

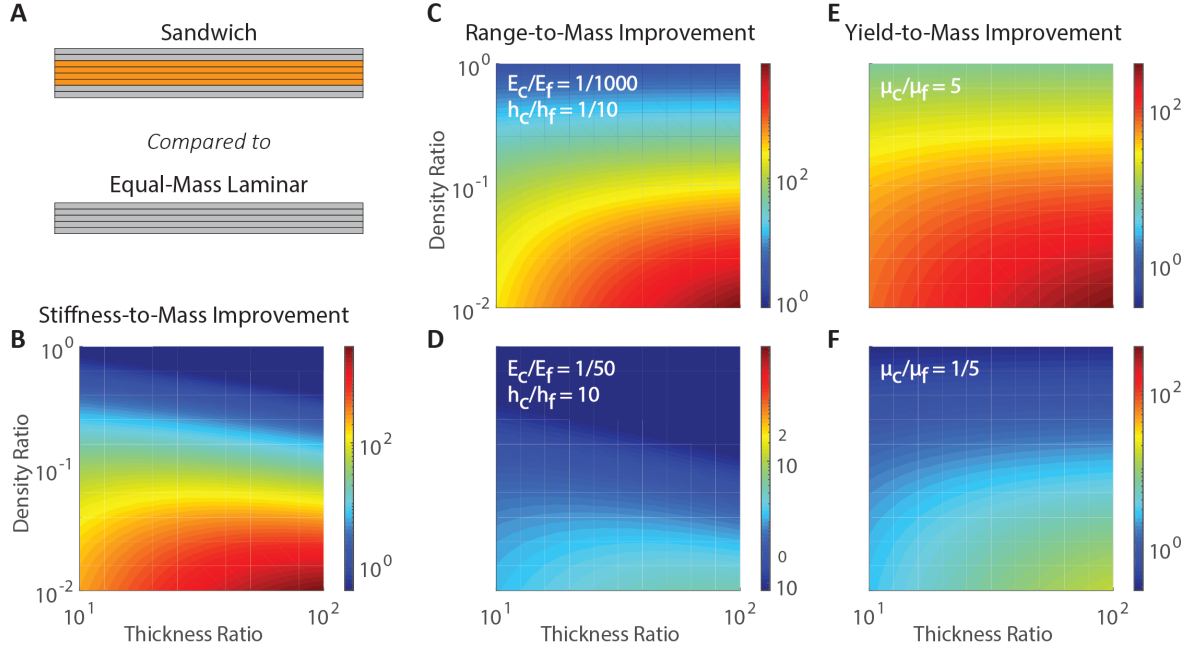


Figure B.2: A) Diagram of an equal-mass laminar jamming structure. B-F) Contour maps illustrating improvement ratios of sandwich jamming structures compared to equal-mass laminar jamming structures. Note that over the examined range of parameters, stiffness-to-mass and range-to-mass can be improved by almost 3 orders of magnitude, whereas yield-to-mass can be improved by 2 orders of magnitude.

$$\left(\frac{F_{crit}}{m}\right)^* = \frac{3}{2} \frac{\mu_c}{\mu_f} \frac{\frac{c}{f}}{\frac{\rho_c}{\rho_f} \frac{c}{f} + 1} \quad (\text{B.66})$$

Note that although the stiffness-to-mass improvement ratios depend only on the density ratio and the total thickness ratio, the range-to-mass and yield-to-mass improvement ratios depend on additional non-dimensional parameters. The range-to-mass improvement ratio also depends on $\frac{E_c}{E_f} \left(\frac{h_c}{h_f}\right)^2$, and the yield-to-mass improvement ratio depends on $\frac{\mu_c}{\mu_f}$.

B.2.2 Contour Maps for Equal-Mass and Equal-Volume Comparisons

Analogous to the contour maps provided in Figure 4.2 for the equal-material comparison, contour maps for the equal-mass and equal-volume comparisons are provided in Figure B.2 and Figure B.3, respectively.

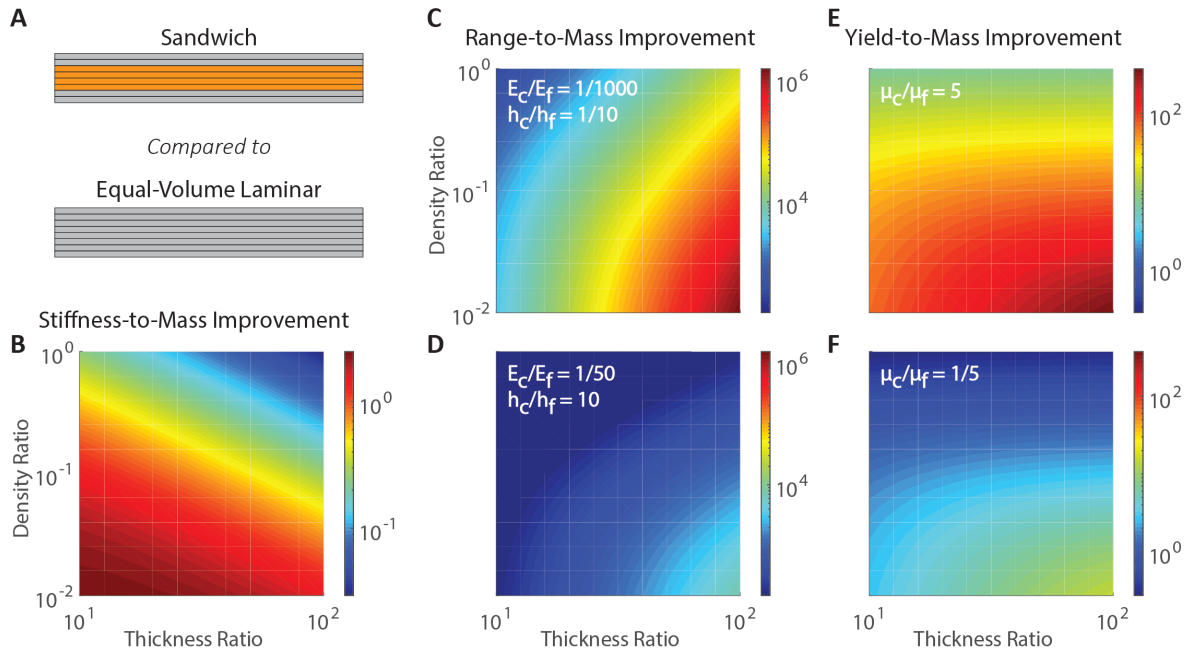


Figure B.3: A) Diagram of an equal-volume laminar jamming structure. B-F) Contour maps illustrating improvement ratios of sandwich jamming structures compared to equal-volume laminar jamming structures. Note that over the examined range of parameters, range-to-mass can be improved by 6 orders of magnitude, whereas yield-to-mass can be improved by 2 orders of magnitude.

B.3 Finite Element Simulations

B.3.1 Procedures and Parameters

To corroborate the predictions of the analytical model, finite element simulations were conducted using commercial simulation software (Abaqus 2017, Dassault Systèmes, Vélizy-Villacoublay, France). Each layer was defined as a 2-dimensional body where the length and thickness were directly modeled and the width was prescribed. For all layers, $L = 150 \text{ mm}$ and $w = 1 \text{ mm}$, where L is the length and w is the width. For all simulations, $\nu_c = \nu_f = 0.25$, where ν_c and ν_f are the Poisson's ratios of the core and face material, respectively.

Frictional contact with a *general contact* formulation was prescribed at the interfaces between the layers. To mitigate elastic slip, a strict elastic slip tolerance of $5 * 10^{-5}$ was prescribed. A static implicit solver was selected, and large deformation analysis was enabled. A uniform mesh of square plane-stress elements with reduced integration (CPS4R) was used, with two elements across the thickness of each face layer (Figure 4.3C). A mesh refinement study was conducted to ensure that a finer mesh was not necessary.

The model was loaded and supported in 3-point bending (Figure 4.3A). For support, $\delta_{supp} = 100 \text{ mm}$, where δ_{supp} is the distance between the support points. For loading, in a first step, pressure (equal to vacuum pressure) was applied as a ramp to all outer surfaces of the model. In a second step, a vertical displacement boundary condition was applied as a ramp to the center node of the topmost surface with $\delta_{load} = 10 \text{ mm}$, where δ_{load} is the maximum loading distance. Automatic timestepping was enabled.

Dimensional and material parameters were varied in order to sufficiently corroborate the analytical results over a wide range of the parameter space. The specific values of these parameters are given in Table B.2. For each simulation, the vertical reaction force and displacement were extracted at the loading point. Corresponding force-versus-deflection curves are given in Figure B.4.

Table B.2: Dimensional and material parameters used in finite element simulations. Brackets denote a set of multiple discrete values that were simulated. Note that c and f were not directly varied, but are simply computed here by the relations $c = n_c h_c$ and $f = n_f h_f$. Also note that in the set of simulations for which $h_c = 0.125$ (i.e., the second row of the table), the number of core values are doubled in order to ensure that the thickness ratio $\frac{c}{f}$ has a consistent range over all simulations.

n_c	n_f	h_c [mm]	h_f [mm]	c [mm]	f [mm]	E_c [MPa]	E_f [MPa]	μ_c	μ_f
{4,5,6,7,8}	4	0.25	0.025	{1,1.25,1.5,1.75,2}	0.1	100	1e4	0.25	0.25
{8,10,12,14,16}	4	0.125	0.025	{1,1.25,1.5,1.75,2}	0.1	50	1e4	0.25	0.25
{4,5,6,7,8}	4	0.25	0.025	{1,1.25,1.5,1.75,2}	0.1	100	1e4	0.5	0.25

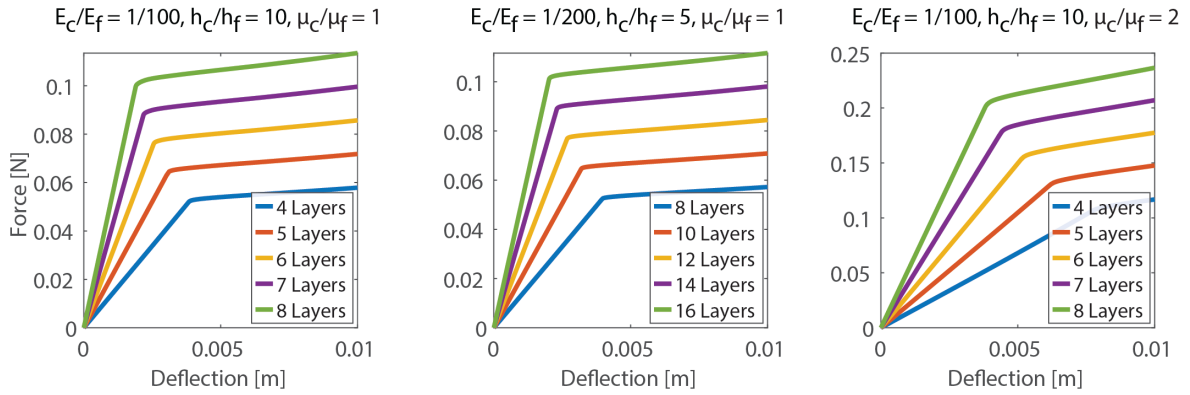


Figure B.4: Force-versus-deflection curves extracted from finite element simulations. These plots were used to calculate performance metrics, which were compared to analytical predictions in Figure 4.3.

B.3.2 A Note on the Comparison to Analytical Predictions

Stiffnesses were derived from the finite element results by taking derivatives of the initially-linear regimes of the force-versus-deformation curves. Strictly speaking, a stiffness derived in this manner has contributions from both the bending stiffness and shear stiffness of the structure. However, in the structures examined, the bending stiffness contributions were assumed to be dominant; thus, the values could be directly compared to analytically-predicted bending stiffnesses. As demonstrated by the exceptionally close match between finite element results and analytical predictions, this assumption was sound.

The preceding assumption can also be analytically justified as follows. From the theory of partial deflections for sandwich beams,

$$w(x) = w_b(x) + w_s(x) \quad (\text{B.67})$$

where $w(x)$ is the deflection of the sandwich beam as a function of the x -coordinate, $w_b(x)$ is the contribution from bending compliance, and $w_s(x)$ is the contribution from shear compliance [83].

If the sandwich beam is subject to a concentrated transverse force P , the deflection at a point $x = a$ can be written as

$$w(a) = \frac{PL^3}{\alpha D} + \frac{PL}{\beta S} \quad (\text{B.68})$$

where L is the length of the beam, D is the bending stiffness (i.e., the constant of proportionality between the bending moment and the curvature), and S is the shear stiffness (i.e., the constant of proportionality between the average shear stress and the average shear strain), and α and β are constants that depend on the boundary conditions. For example, for the deflection at the midpoint of a beam in 3-point bending, $\alpha = 48$ and $\beta = 4$, and for the deflection at the end of a beam in cantilever tip loading, $\alpha = 3$ and $\beta = 1$.

For bending deflections to be dominant (i.e., shear deflections to be negligible),

$$\frac{\frac{L^3}{\alpha D}}{\frac{L}{\beta S}} \gg 1 \quad (\text{B.69})$$

For sandwich beams, $D = \frac{E_f \frac{f}{2} (c + \frac{f}{2})^2}{2}$ and $S = \frac{G_c (c + \frac{f}{2})^2}{c}$, where G_c is the shear modulus of the core. For linearly elastic materials, $G = \frac{E}{2(1+\nu)}$. Substituting these expressions into (B.69) and simplifying,

$$\frac{\beta}{\alpha} \frac{E_c}{E_f} \frac{2}{1 + \nu_c} \frac{L^2}{cf} \gg 1 \quad (\text{B.70})$$

Among the structures examined in our finite element simulations, the left-hand side of the above equation has a minimum value of 75. Thus, bending stiffness is dominant.

B.4 Optimization

B.4.1 Software Routine

A software routine to optimize sandwich jamming structures was constructed in mathematical analysis software (MATLAB 2018a, MathWorks); a flow chart of the routine is shown in Figure B.5. First, the routine imports a list of materials and material properties that are provided by the user. For each material, the user must specify the elastic modulus E , the density ρ , the coefficient of friction μ , and the minimum material thickness h_{min} , as materials are not practically available in arbitrarily small thicknesses.

Next, the software routine cycles through each possible material pair and determines which pairs satisfy the assumptions of sandwich theory and recommended construction guidelines (i.e., $E_c \ll E_f$). For every satisfactory pair, the routine optimizes each performance metric (i.e., stiffness-to-mass, range-to-mass, and yield-to-mass) separately. The optimization algorithm is a constrained nonlinear algorithm based on gradient descent (*fmincon*). The constraints are mass and volume constraints specified by the user, as well as the h_{min} constraint described above. The optimized parameters are geometric variables (i.e., n_c , n_f , h_c , h_f , c , and f), and the cost functions are simply the reciprocals of the stiffness-to-mass, range-to-mass, and yield-to-mass expressions for sandwich jamming structures that were derived in [Analytical Modeling](#).

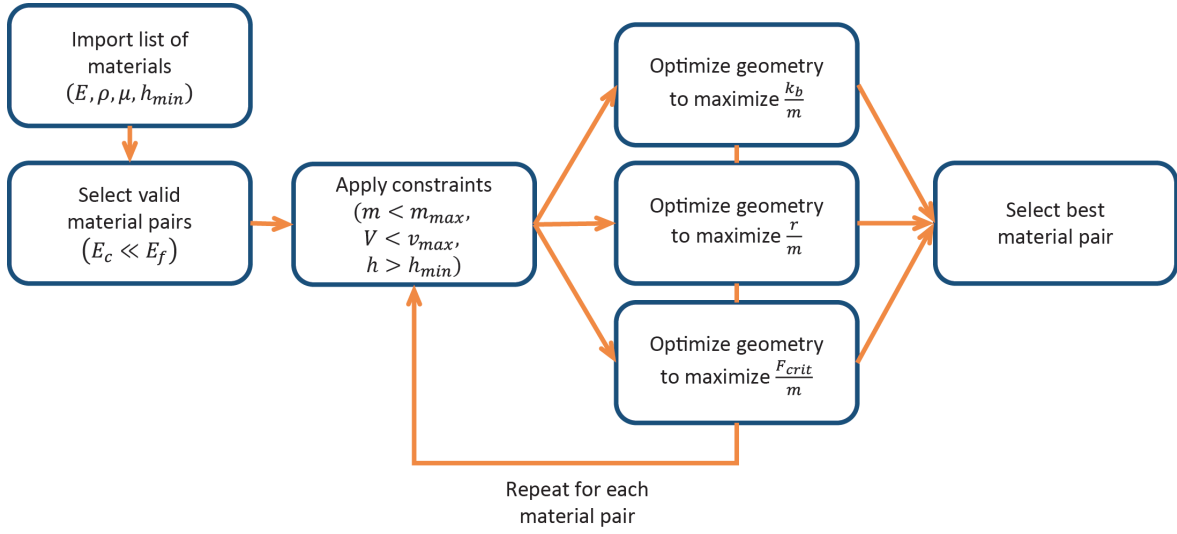


Figure B.5: Flow chart of critical steps in software routine for optimizing sandwich jamming structures.

Table B.3: Materials and material properties used in the optimization case study.

Material	Input Parameter			
	$\rho[\frac{kg}{m^3}]$	$E[MPa]$	μ	$h_{min}[mm]$
Steel	8100	1.73e5	0.38	0.05
Paper	800	4.6e3	0.65	0.1
LDPE	840	2.8e2	0.18	0.1
PU Foam	480	4	0.38	0.79

B.4.2 Additional Data

Table B.3 lists the materials and material properties that were used in the optimization case study. The material properties were derived from experimental measurements, as well as reference values available in online databases. The minimum thicknesses were defined by what was available for a low cost from an online mechanical parts vendor (i.e., McMaster-Carr [106]). Table B.4 and Table B.5 provide the optimization results for the stiffness-to-mass and yield-to-mass ratios, respectively.

Table B.4: Results from optimization case study for stiffness-to-mass ratio. The first- and second-best performing material configurations were steel-foam and steel-paper, respectively.

Material Configuration	Optimized Parameter		
	$c[m]$	$f[m]$	Stiffness-to-Mass $[\frac{N*m^2}{kg}]$
Steel-Paper	0.0070	4.7e-4	1.1e9
Steel-LDPE	0.0071	4.3e-4	1.0e9
Steel-Foam	0.0068	6.8e-4	1.7e9
Paper-Foam	0.0068	6.8e-4	1.1e8
LDPE-Foam	0.0068	6.8e-4	6.3e6

Table B.5: Results from optimization case study for yield-to-mass ratio. The first- and second-best performing material configurations were paper-foam and LDPE-foam, respectively.

Material Configuration	Optimized Parameter		
	$c[m]$	$f[m]$	Yield-to-Mass $[\frac{N}{kg}]$
Steel-Paper	0.0074	1.0e-4	1.4e3
Steel-LDPE	0.0074	1.0e-4	380
Steel-Foam	0.0074	1.0e-4	1.3e3
Paper-Foam	0.0073	2.0e-4	1.6e3
LDPE-Foam	0.0073	2.0e-4	1.5e3

B.5 Demonstrations

B.5.1 Optimization and Fabrication

Prior to fabrication of the orthosis, the optimization software routine was used to determine the configuration of the sandwich jamming structure that would be integrated into the device. The materials considered were the same as those in the optimization case study; however, the minimum layer thickness for steel was adjusted to 0.025 mm ($0.001''$) upon purchase of thin shim stock (9011K211, McMaster-Carr). A maximum mass constraint of 40 g was applied. In addition, in order to achieve comfort and security, the width and length of the laminar jamming structure were selected to be 25 mm and 150 mm , respectively; a maximum height constraint of 3.5 mm was then applied.

Optimized steel-paper sandwich jamming structures were found to provide an excellent balance of stiffness-to-mass, range-to-mass, and yield-to-mass behavior compared to other material configurations. The optimized structures consisted of 6 layers of 0.025 mm ($0.001''$)-thick low-carbon steel, 30 layers of 0.1 mm ($0.004''$)-thick paper, and 6 layers of steel. The wrist orthosis itself consisted of an elastic hand sleeve made of a copper-nylon composite material (Copper Infused Wrist Sleeve, UptoFit Sports, Wilmington, DE) that was sewn to an arm sleeve constructed from a fabric-foam composite material with a non-slip interior coating (NuStim, Fabrifoam, Exton, PA).

B.5.2 Human Subject Testing

All experimental procedures were approved by the Harvard University Committee on the Use of Human Subjects. Two healthy adult female participants were recruited for the study.

Isometric Hold Task

Subjects were asked to complete the isometric hold tasks with no orthosis and with two orthosis configurations: a sandwich-jamming orthosis in the unjammed state and a sandwich-jamming orthosis in the jammed state. Two trials were conducted for each of the three configurations.

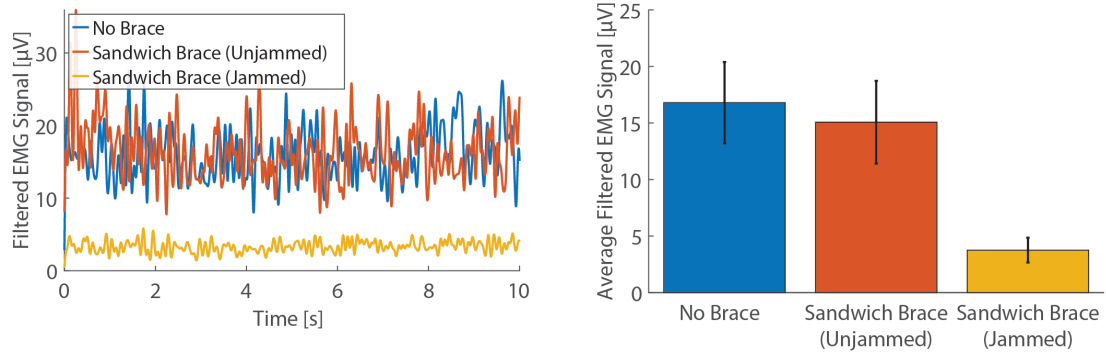


Figure B.6: Comparison of EMG profiles for a second human subject. EMG profiles are shown during the no-brace, unjammed, and jammed conditions. The average EMG signal for the unjammed condition was slightly less than that of the no-brace condition, and the average signal for the jammed condition was approximately 75% less than that of the no-brace condition. Error bars on the bar plot denote ± 1 standard deviation.

In each trial, a 1 *kg* weight was suspended from the hand5, and muscle activation data was recorded for 10 *s* using a portable surface electromyography system (TeleMyo 2400T G2, Noraxon U.S.A., Scottsdale, AZ) recording at 1500 *Hz*.

After the tests were conducted, the raw data was filtered using mathematical analysis software (MATLAB 2018a, MathWorks). First, a band-stop filter was applied between 58 *Hz* and 62 *Hz* to mitigate power-line noise. Next, a bandpass filter was applied with cutoff frequencies of 10 *Hz* and 350 *Hz*. The data was then rectified, and an envelope was fit to the resulting curves. Figure 4.5D-E showed EMG data for the isometric hold task for one human subject; Figure B.6 shows analogous data for a second subject.

Range-of-Motion Test

Subjects were asked to complete the range-of-motion tests with no orthosis and a sandwich jamming orthosis in the unjammed state. Subjects were asked to flex and extend their wrists 5 times to the maximum angle that they still perceived as comfortable while remaining relaxed. A goniometer was used to manually measure the maximum flexion and extension angles during each cycle.